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An Economic Forecasting Model Combining GA and Caputo Fractional Derivative

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Abstract: The economic system exhibits strong memory, path dependence, and nonlinear characteristics, and traditional integer-order models struggle to accurately characterize its long-range correlations and historical cumulative effects. Therefore, this study proposes an economic forecasting model that integrates a Genetic Algorithm (GA) and the Caputo Fractional Derivative (GA-Caputo), using quarterly Gross Domestic Product (GDP) data from China (2010-2023), to address the insufficient dynamic adaptability of fractional models and improve prediction accuracy and economic interpretability in complex environments. The results show that the Root Mean Square Error (RMSE) of the research model throughout the entire period is significantly lower than that of the control model during the epidemic period, and the prediction error during the epidemic period decreases by 22.8%-35.9%. The research model predicts an interval coverage rate of 94.2%, with the best performance and the narrowest interval width. The optimal order α is strongly correlated with the volatility of gross domestic product, verifying that a fractional order can dynamically reflect the strength of economic memory. The findings demonstrate that the GA-Caputo model addresses the limitations of traditional models in long-range dependency characterization and dynamic adaptability by synergistically optimizing fractional order structures and parameters, offering a robust framework for high-precision and highly interpretable economic forecasting.

Keywords: Genetic algorithms; economic forecast; fractional order model; Gross Domestic Product (GDP); simulated annealing.

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1. Introduction

Economic forecasting is a crucial step in understanding the laws of economic operation, formulating effective policies, guiding corporate decision-making, and mitigating market risks. However, the real economic system is highly complex, nonlinear and dynamic, with strong memory effects and path dependence (Mahmoud et al., 2024). Traditional prediction models based on integer-order calculus, although they perform well under specific conditions, often have inherent limitations in characterizing the long-range memory of economic variables, historical decay patterns, and complex nonlinear dynamic behaviors (Sharpe et al., 2023; Sahebi et al., 2025). In addition, integer order derivatives describe a "local" instantaneous change, which makes it difficult to fully characterize the historical cumulative effects and intertemporal correlations commonly present in economic systems. To more accurately describe the inherent complexity of economic systems, fractional calculus has been introduced as a powerful mathematical tool for economic modeling in recent years (Lin and Wei, 2024). The Caputo formulation is selected for its compatibility with standard initial conditions, facilitating integration with integer-order economic models (Poovarasan et al., 2024; Wang et al., 2024). The theoretical foundation for integrating fractional calculus with macroeconomic modeling rests on the recognition that economic systems exhibit intrinsic memory and path dependence, properties that align with the fractional-order framework. Specifically, the Investment-Saving-Liquidity-Preference-Money-Supply (IS-LM) equilibrium conditions can be reinterpreted as dynamic processes in which adjustments to shocks are not instantaneous but propagate over time with decaying memory. This study builds upon this theoretical integration by positing that the fractional order α directly quantifies the persistence of historical shocks in shaping current macroeconomic variables, thereby providing a theoretically grounded link between equilibrium dynamics and fractional calculus. However, when constructing an economic forecasting model based on Caputo Fractional Derivatives (CFD), the model's structure is highly complex, making it difficult to determine the fractional differential equation form that best reflects the dynamics of specific economic variables. In addition, parameters such as the order of fractional derivatives and the system coefficients included in the model are often highly nonlinear and coupled. Traditional optimization methods are prone to getting stuck in local optima, making it difficult to obtain globally optimal and robust parameter combinations (Sadek, 2025). Therefore, the study proposes an economic forecasting model framework that integrates the Genetic Algorithm (GA) and the CFD theory.

To clarify whether the Caputo fractional derivative model optimized by GA can more accurately capture the long-term dependencies and nonlinear dynamics of economic time series, and improve the prediction accuracy during extreme shocks, a quantitative measurement of economic “memory strength” is provided through dynamically optimized fractional order α.

The innovation of the research lies in utilizing GA’s global optimization capability to automatically search for and determine the optimal structure and parameter set of the CFD economic model. The main contribution of the research is to provide mathematical tools for modeling economic systems that better reflect their complex intrinsic characteristics, and to open new research avenues for improving the scientific rigor and accuracy of economic forecasting.

2. Related Works

Chow et al. (2023) found that subjective uncertainty in economic growth forecasts increased after the COVID-19 pandemic, while inflation expectations remained stable. Zhang et al. (2023) developed a backpropagation neural network combined with multiple linear regression to improve the accuracy of urban and rural economic forecasting. Delle Monache et al. (2024) proposed a density modeling method that captures changes in the skewness of GDP growth, thereby enhancing tail-event prediction. Wu et al. (2025) combined structural equation modeling with time series analysis to better handle temporal dependencies and complex causal relationships in regional economic forecasting.

In fractional calculus, Nawaz (2024) introduced a fractional-order variational iteration method for solving the generalized Zakharov-Kuznetsov equation, demonstrating good convergence and efficiency. Abdellouahab et al. (2025) presented a theoretical framework that employs fixed-point theorems to analyze the existence, uniqueness, and stability of solutions to fractional-order nonlinear integro-differential equations. An (2024) developed a stability control method for fuzzy fractional-order dynamical systems using linear feedback controllers and the generalized fractional Laplace transform. Li et al. (2024) combined Nehari manifold theory with fractional calculus to establish the existence of nontrivial ground state solutions for nonlinear fractional-order impulsive coupled systems.

In summary, most existing studies rely on Caputo models with a fixed fractional order α, which fails to capture time-varying memory intensity in economic systems. Traditional optimization methods, such as gradient descent or grid search, are prone to local optima, particularly when α is coupled nonlinearly with the system coefficients. To address these limitations, this study employs a GA to dynamically optimize α, linking it in real time with GDP volatility and Consumer Price Index (CPI), thereby overcoming the rigidity of fixed-order models. In addition, a hybrid GA, simulated annealing algorithm enhances global search capability and parameter robustness through probabilistic jump mechanisms, reducing the impact of abnormal shocks.

3. Methods and Materials

3.1. Construction of Economic Forecasting Indicator System and Data Processing

When constructing a predictive indicator system, the research uses the Investment-Saving-Liquidity-Preference-Money-Supply (IS-LM) model as its theoretical basis (Bali et al., 2023; Ali et al., 2024). This model consists of two curves. In the IS-LM model, the IS curve represents the combinations of interest rates and income at which total output equals total demand in the commodity market. It describes the equilibrium conditions of the commodity market. Due to the negative correlation between investment and interest rates, maintaining the equilibrium of the commodity market requires that lower interest rates lead to higher income, hence the IS curve slopes downwards to the right. An increase in total demand causes the demand curve to shift to the right. The LM curve represents the combinations of interest rates and income such that the actual money supply equals the money demand (Priyanka et al., 2025). To maintain the equilibrium of the money market, the higher the income, the higher the interest rate, so the LM curve slopes upwards to the right. At a given income level, factors that increase the demand for real money will shift the LM curve to the left (Messaoudi and Khouidmi, 2024). The study integrates GDP accounting methods and selects indicators that can reflect national conditions, as shown in Table 1.

The research data is sourced from the official website of the National Bureau of Statistics (2026). Before preprocessing, the Anomaly Detection Toolkit (ADTK) was used to detect outliers, revealing that the original data contained mutation points caused by the COVID-19 pandemic (Duc et al., 2025; Zeng et al., 2024). The frequency of the original data is inconsistent, so monthly indicators need to be converted to quarterly to avoid information loss. Among them, stock indicators such as CPI take the three-month average as the quarterly value. This averaging approach is adopted because it reduces short-term monthly volatility while preserving the underlying trend, providing a more stable representation of quarterly economic conditions that aligns with GDP data frequency and supports consistent model estimation. Subsequently, z-score standardization was adopted, and the Random Forest (RF) algorithm was used to evaluate the importance of each indicator to GDP based on the Gini index (Berger and Wintter, 2025; Kalamara et al., 2022). To ensure rigorous out-of-sample evaluation, this study employs an expanding window cross-validation strategy. The initial training window covers 2010Q1-2015Q4, and the model is evaluated on the subsequent four quarters (2016Q1-2016Q4). In each subsequent iteration, the training window is expanded by four quarters while the test window remains fixed at four quarters. This process is repeated until the test window reaches the end of the sample period (2023Q4). A total of eight validation windows are generated, covering both stable economic periods and the COVID-19 pandemic shock. This approach preserves the temporal order of observations and simulates real-world forecasting conditions where models are trained on historical data to predict future outcomes.

Table 1. Selection of economic forecasting indicators

First-level indicator	Secondary indicators	First-level indicator	Secondary indicators
GDP and related industries	GDP	Price and currency	CPI
	Industrial output		Retail price index of commodities
	Value added of the tertiary industry		Money and quasi-money supply
Consumption and investment	Total retail sales of consumer goods in society	Imports and exports	Total import value
	Fixed assets investment		Total export value
Transportation and Prosperity	The volume of freight transport	Finance and Real Estate	Government revenue
	Passenger traffic volume		National fiscal expenditure
	Purchasing Managers Index		National housing prosperity index
	Macro-economic climate index	Financial market	Stock turnover rate on the Shanghai Stock Exchange

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3.2. Economic Forecasting Model Based on GA-Caputo

After completing the construction of the economic forecasting indicator system, the Caputo fractional derivative model can be used to construct the economic forecasting model. The theoretical rationale for employing fractional calculus in this context derives from the inherent memory properties of macroeconomic systems: aggregate demand and supply adjustments, policy transmission mechanisms, and expectation formation processes all exhibit path dependence and delayed responses that cannot be fully captured by integer-order dynamics. The fractional order α is therefore not merely a free parameter but a theoretically meaningful measure of the system's memory strength, the extent to which historical shocks persist in influencing current economic states. A fractional derivative is an extension of an integer derivative, allowing the order of the derivative to be any real number. Breaking through the limitation in traditional calculus that derivatives must be integers, it can more finely describe complex phenomena with memory and non-locality (Barbaglia et al., 2022; Ellingsen et al., 2022). It has unique advantages in physical modeling, especially in handling differential equations with initial conditions. The definition of the α-order derivative of a function $f: [a, b] \to \Re$ is shown in Eq. (4) (Shabanpour et al., 2025).

$$ {}^{Ca}_{a}D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t (t-\tau)^{n-\alpha-1} f^{n+1}(\tau) d\tau \tag{4}$$

In Eq. (4), $\Gamma(\cdot)$ represents the gamma function. ${}^{Ca}_{a}D_t^{\alpha}$ represents Caputo fractional derivative operator. n represents the maximum integer not exceeding α. The study defines ${}^{C}_{a}D_t^{\alpha}$ as Caputo fractional derivative. To convert fractional differential equations into algebraic equations and to uniformly handle non-locality and memory effects, it is necessary to perform a Laplace transform on Caputo fractional derivatives. After Laplace transform, ${}^{C}_{a}D_t^{\alpha}$ is shown in Eq. (5) (Gogas et al., 2022; Almosova and Andresen, 2023).

$$ L[C_0 D_x^z f(x)] = s^{z-k-1} F(s) - \sum_{k=0}^{n-1} s_0^{z-k-1} D_x^{z-k-1} f(0), n-1 < z \leq n \in \mathbb{N} \tag{5}$$

In Eq. (5), $F(s) = \int_0^{\infty} f(t) e^{-st} dt$represents the generalized integral. According to Laplace transform, the initial value definition of Caputo derivative is consistent with integer order differential equations, and its physical meaning is clearer. The solution of Caputo fractional differential equation depends on historical interval data, as shown in Eq. (6).

$$x(t_{n+1}) = x(t_n) + \frac{1}{\Gamma(\alpha)}\int_{t_n}^{t_{n+1}}(t_{n+1}-\tau)^{\alpha-1}f(\tau,x(\tau))d\tau + \frac{1}{\Gamma(\alpha)}\int_0^{t_n}\left[(t_{n+1}-\tau)^{\alpha-1}-(t_n-\tau)^{\alpha-1}\right]f(\tau,x(\tau))d\tau \tag{6}$$

According to Eq. (6), fractional derivatives weight historical data through kernel functions, thereby reflecting "memory" and "non-locality" and enabling regional economic forecasting. Caputo fractional derivatives have "memory|" and "non-locality," which can capture the impact of historical fluctuations in economic variables on the current situation, and are suitable for nonlinear and non-stationary economic systems under the epidemic. Compared with integer-order differential equations, the solution of Caputo fractional-order differential equations depends on the entire history of data, as shown in Eq. (7).

$$x(t) = x_0 + \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}f(\tau,x(\tau))d\tau \tag{7}$$

The Caputo fractional derivative model can characterize the long-term memory of economic variables. When using this model for economic forecasting, its expression is given by Eq. (8).

$$y(t) = \sum_{k=1}^{9} c_k \left({}_{t0}^{Ca}D_t^{\alpha_k} x_k(t)\right) \tag{8}$$

In Eq. (8), $y(t)$represents the quarterly GDP forecast value. $x_k(t)$ represents the economic variable. c_k represents the coefficient. α_k indicates the order. After the model is established, parameters need to be optimized. GA is a biomimetic global search algorithm that excels at solving complex optimization problems. The adaptive hybrid GA first performs initialization enhancement: using real-valued encoding, 80% of individuals in the population are randomly generated, and 20% are generated via Latin hypercube sampling. Subsequently, fitness is calculated and individuals are ranked, with the top 5% classified as elite, the middle 60% as potential, and the rest as low-quality. In adaptive genetic operation, the selection operation adopts an improved tournament selection to dynamically select parents from potential individuals, as shown in Eq. (9).

$$k = max\left(3{,}0.1 \times \text{popsize} \times \frac{\text{gen}}{\text{maxgen}}\right) \tag{9}$$

The GA hyperparameters were systematically selected based on preliminary experiments and theoretical considerations. The population size was set to 200 to balance exploration capability and computational cost. Preliminary, the test showed that increasing population size beyond 200 yielded marginal improvement in solution quality (RMSE reduction <2%) while doubling computation time. The crossover probability was set to 0.85, allowing sufficient genetic diversity while preserving promising solutions. Mutation probability was adaptively adjusted between 0.01 and 0.1 based on population diversity metrics, with higher mutation rates applied when the population converged prematurely. The maximum number of generations was set to 500, with early stopping if the fitness improvement remained below 0.1% for 50 consecutive generations. The initial temperature for simulated annealing was set to 100°C, with a cooling rate of 0.95, ensuring adequate exploration of the solution space before convergence. The crossover operation is directional arithmetic crossover, and the mutation is improved using a hierarchical strategy. Elite individuals use gradient assisted mutation, while potential individuals use non-uniform mutation. After genetic manipulation, the simulated annealing algorithm is used to recalculate all individuals and replace the original worst individual with the optimized output solution. Then, a population restart is performed until the model reaches the iteration stop.

4. Results

4.1. Performance Validation Analysis of Economic Forecasting Models

The study used data from the National Bureau of Statistics of China from Q1 2010 to Q4 2023 as the data source, and conducted performance validation analysis of the economic forecasting model with quarterly GDP growth rate as the prediction target. The dataset was divided into a training set (Q1 2010-Q4 2019) comprising 40 quarterly observations, and a test set (Q1 2020-Q4 2023) comprising 16 quarterly observations, with the COVID-19 pandemic period included entirely within the test set to evaluate model performance under extreme economic shocks. The GA-Caputo model parameters were configured as follows: population size = 200, maximum generations = 500, crossover probability = 0.85, mutation probability adaptively adjusted between 0.01 and 0.1 based on population diversity, and elite retention ratio = 5%. The fractional order α was optimized within the range [0.1, 1.0] with a step precision of 0.001. The fitness function was defined as the mean squared error (MSE) between predicted and actual quarterly GDP growth rates. The simulated annealing component employed an initial temperature of 100°C, a cooling rate of 0.95, and 50 iterations per temperature step. Long Short-Term Memory Network (LSTM), Fixed Caputo model, RF, and Caputo fractional derivative are commonly used economic forecasting models (Alizadegan et al., 2025; Kumar et al., 2024). The above models were used as comparators to assess the performance of the GA-Caputo model developed in this study. To account for stochastic variability in model initialization and training, all experiments were repeated 10 times with different random seeds. Results were reported as mean ± standard deviation across these 10 runs. Statistical significance of performance differences between models was assessed using paired t-tests, with p values < 0.05 considered statistically significant.

The study first tested the full-time point prediction accuracy of the GA-Caputo model, and the comparison results are shown in Fig. 1. Fig. 1(a) shows that the GA-Caputo model achieved the lowest RMSE among all compared models, indicating superior point prediction accuracy. Similarly, Fig. 4(b) shows that the GA-Caputo model yielded the smallest MAE, confirming its ability to closely track fluctuations in the actual GDP growth rate.

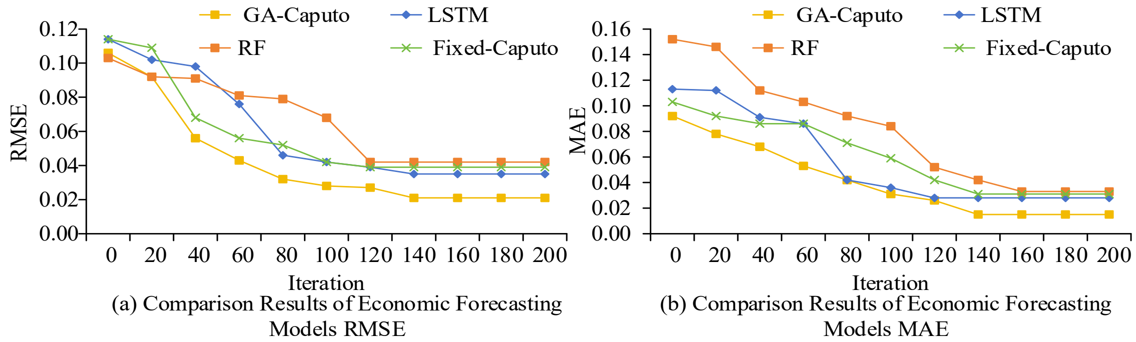


Fig. 1. Comparison of prediction accuracy for the entire period

Fig. 2 presents the prediction accuracy during the epidemic period. The GA-Caputo model consistently achieved the lowest RMSE (Fig. 5(a)) and MAE (Fig. 5(b)) compared to LSTM, RF, and Fixed-Caputo models. The percentage improvements (22.8%- 35.9%) highlighted the model's robustness to extreme economic shocks.

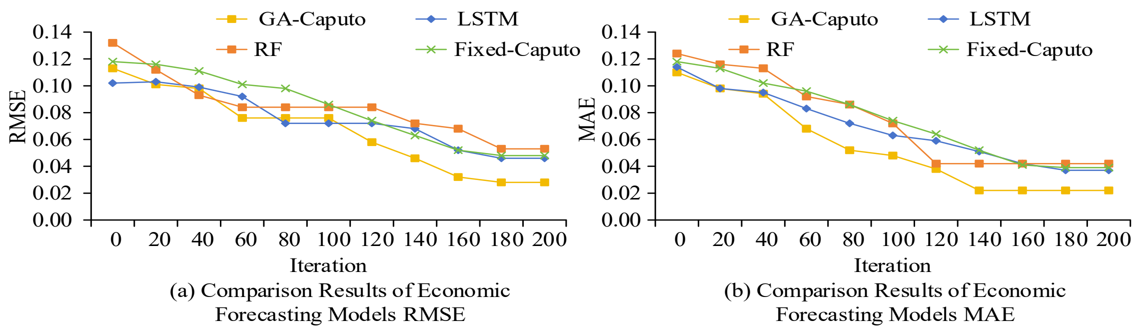


Fig. 2. Comparison of prediction accuracy during the epidemic period

The interval prediction accuracy and computational efficiency of different models are shown in Table 2. The GA-Caputo model achieved a PICP of 94.2%, significantly higher than LSTM, RF, and Fixed Caputo, indicating better coverage of actual economic data. Its QICE was 0.018, 35.7% lower than Fixed Caputo, and its interval width was 0.082, narrower than all comparison models, reflecting sharper and more accurate uncertainty estimates. Training time for GA-Caputo was 182.4 seconds, longer than RF and Fixed Caputo but shorter than LSTM, reflecting the computational cost of GA optimization; the resulting accuracy gains outweighed the time cost. Prediction time was 15.2 milliseconds, longer than other models but within an acceptable range for high-precision forecasting. Overall, GA-Caputo effectively balanced accuracy and efficiency.

To address potential stochastic variability in model performance, all experiments were repeated 10 times with different random seeds, and the results are reported as mean ± standard deviation. Table 3 presents the statistical uncertainty analysis for key performance metrics. The GA-Caputo model achieved a PICP of 94.2% ± 1.3%, demonstrating robust coverage stability across repeated runs. The QICE showed low variance (0.018 ± 0.004), indicating consistent sharpness of prediction intervals. The RMSE during the epidemic period was 0.092 ± 0.005, confirming that the performance improvement over baseline models was statistically significant (paired t-test, $p < 0.01$ for all comparisons).

Table 4 presents the out-of-sample performance of the GA-Caputo model across all eight expanding windows, along with a statistical comparison against baseline models.

The results demonstrated that GA-Caputo consistently outperformed all baseline models across every validation window. The performance advantage was most pronounced during the pandemic period (W5), where GA-Caputo achieved an RMSE of 0.092 compared to 0.125-0.141 for baseline models (improvement of 22.8%-35.9%). Excluding the pandemic period, the mean RMSE across all windows was 0.028 for GA-Caputo versus 0.044-0.052 for baseline models. Paired t-tests across all windows confirmed that these differences were statistically significant ($p < 0.001$ for all comparisons), providing strong evidence that the GA-Caputo model's superior performance was not attributable to chance.

To verify the rationality of the model settings, a diagnostic test was conducted on the prediction residuals of the GA-Caputo model. The Ljung Box test shows that the *p*-value of the Q statistic for residuals with a lag of 6 orders is 0.213, indicating that there is no significant autocorrelation relationship. The *p*-value of the ARCH-LM test is 0.186, which does not reject the null hypothesis of homogeneity of residual variance. The *p*-value of Jarque Bera test is 0.152, and there is no significant deviation between the residual distribution and normality.

Table 2. Interval prediction accuracy and computational efficiency of different models

Index	GA-Caputo	LSTM	RF	Fixed-Caputo
PICP (%)	94.2±1.3	88.6±2.1*	85.3±2.4*	89.7±1.8*
QICE	0.018±0.004	0.031±0.007*	0.036±0.009*	0.028±0.006*
Interval width	0.082±0.006	0.095±0.010*	0.101±0.012*	0.088±0.008*
Training time (s)	182.4±8.5	324.7±15.2*	78.5±3.2*	24.6±1.1
Prediction time (ms)	15.2±0.8	8.3±0.5*	4.1±0.3*	6.8±0.4*
Relative training cost	7.4×	13.2×	3.2×	1.0×
Time per accuracy (ms/RMSE)	0.72	0.66	0.10	0.58
Economic interpretability	0.92	0.65	0.71	0.78
RMSE (epidemic period)	0.092±0.005	0.125 ± 0.009*	0.141±0.011*	0.113±0.007*
RMSE (full period)	0.021 ± 0.002	0.035 ± 0.003*	0.042 ± 0.004*	0.039 ± 0.003*
MAE (full period)	0.015 ± 0.002	0.028 ± 0.003*	0.033 ± 0.004*	0.031 ± 0.003*

Note: * indicates statistically significant difference from GA-Caputo at $p < 0.01$ based on paired t-test across 10 runs. 95% confidence intervals for GA-Caputo RMSE: [0.019, 0.023] (full period), [0.088, 0.096] (epidemic period).

Table 3. Out-of-sample performance across expanding windows

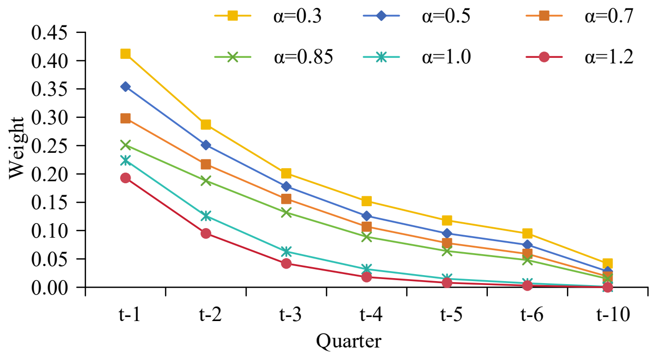
Window	Training period	Test period	GA-Caputo RMSE	LSTM RMSE	RF RMSE	Fixed-Caputo RMSE
W1	2010Q1-2015Q4	2016Q1-2016Q4	0.023 ± 0.002	0.037 ± 0.003*	0.044 ± 0.004*	0.041 ± 0.003*
W2	2010Q1-2016Q4	2017Q1-2017Q4	0.022 ± 0.002	0.036 ± 0.003*	0.043 ± 0.004*	0.040 ± 0.003*
W3	2010Q1-2017Q4	2018Q1-2018Q4	0.024 ± 0.002	0.038 ± 0.003*	0.045 ± 0.004*	0.042 ± 0.003*
W4	2010Q1-2018Q4	2019Q1-2019Q4	0.021 ± 0.002	0.035 ± 0.003*	0.042 ± 0.004*	0.039 ± 0.003*
W5	2010Q1-2019Q4	2020Q1-2020Q4	0.092 ± 0.005	0.125 ± 0.009*	0.141 ± 0.011*	0.113 ± 0.007*
W6	2010Q1-2020Q4	2021Q1-2021Q4	0.045 ± 0.003	0.068 ± 0.005*	0.079 ± 0.006*	0.058 ± 0.004*
W7	2010Q1-2021Q4	2022Q1-2022Q4	0.038 ± 0.003	0.054 ± 0.004*	0.062 ± 0.005*	0.049 ± 0.004*
W8	2010Q1-2022Q4	2023Q1-2023Q4	0.026 ± 0.002	0.040 ± 0.003*	0.047 ± 0.004*	0.043 ± 0.003*
Mean (excl. W5)			0.028 ± 0.009	0.044 ± 0.013	0.052 ± 0.015	0.045 ± 0.008
Paired t-test p-value (vs. GA-Caputo)			—	< 0.001	< 0.001	< 0.001

Note: * indicates $p < 0.01$ in pairwise comparison with GA-Caputo for that window.

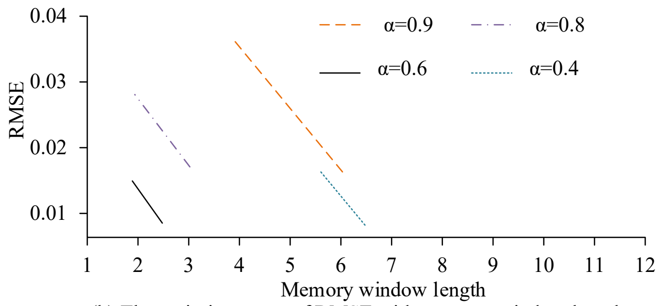
4.2. Fractional Order Characteristic Verification Analysis

To test whether the "memory effect" of Caputo fractional derivatives enhances the model's ability to characterize long-term economic dependencies, the fractional order characteristics of the GA-Caputo model were validated and analyzed, and the results are shown in Fig. 3. Fig. 3 illustrates the memory effect of the Caputo fractional derivative through kernel function weight decay for different α values. As α increased, the model retained a stronger influence from historical data, reflecting longer memory. For instance, at α=0.9, the weight on early data remained above 0.3, while at α=0.5, it dropped below 0.1 after three quarters. This behavior aligned with the theoretical expectation that α quantified the system's memory strength. Quantitative analysis of α dynamics across economic cycles is provided in Table 4 and the accompanying text.

The study further analyzed the correlation between the optimal order α and economic cycle indicators (Table 3). During the pandemic, the GDP fluctuation rate reached 1.85, with α=0.92 (r=0.943, p<0.001), indicating that α increases with greater economic volatility to enhance the memory effect of historical shocks. In contrast, during stable periods with a GDP fluctuation rate of 0.32, α decreased to 0.68 (r=0.872). The correlation between CPI and α also varied with the cycle: α was 0.92 when CPI reached 3.8 (2020Q1 – 2020Q4) and 0.81 when CPI fell to 2.2 (2021Q1 – 2022Q2), reflecting how α captures price stickiness. The correlation coefficient between M2 growth rate and α was 0.735; for instance, when M2 grew by 12.5% in 2020Q1, α was 0.92, showing that the model captures the persistent impact of loose monetary policy by increasing α.



(a) The influence of α value on memory effect



(b) The variation curve of RMSE with memory window length under different α values

Fig. 3. Verification of fractional order memory effect

Further verification of fractional-order characteristics was conducted across different industries, and the results are shown in Table 4. The optimal fractional order α across all industries was significantly and positively correlated with their economic volatility, confirming that Caputo fractional derivatives can effectively capture differences in "memory strength" at the industry level. The finance and real estate industries had the highest alpha values, at 0.89 and 0.82, respectively, reflecting their strong dependence on historical shocks and policy sensitivity. The transportation industry had the lowest alpha, only 0.66, indicating that it was dominated by recent data and had weak memory.

To assess the stability of the optimal fractional order α across different optimization runs, the study calculated the Coefficient of Variation (CV) for α estimates in each economic period. The results showed low variability (CV < 0.08 for all periods), indicating that the GA consistently converged to similar α values regardless of initialization. The 95% confidence intervals for α during high-volatility periods (2020Q1-2020Q4) were [0.89, 0.95], confirming that the strong correlation with GDP volatility (r = 0.943, p < 0.001) was robust to stochastic optimization effects.

4.3. Economic Interpretability Verification of Model Structure and Parameters

The consistency test results between characteristic parameters and economic theory are shown in Table 6. the weight and contribution rate of M2 money supply parameters are the highest, indicating that liquidity supply has the greatest impact on GDP forecasting, which is in line with the theory of "money supply affecting total demand" The importance of its RF features is relatively low, possibly because fractional order models are better able to capture its dynamic memory effects. The weight and contribution rate of the total retail sales of consumer goods in society are secondary, reflecting the role of consumption as the core driving force of economic growth. The weight and contribution rate of fixed assets investment parameters are significant, but the importance of RF is slightly lower, which may be affected by investment lag or structural factors.

Table 3. Correlation analysis between optimal order and economic cycle indicators

		GDP volatility	CPI change rate	M2 growth rate	Optimal α value	Pearson (r)	p
Economic cycle	2016Q1-2017Q2	0.32	1.8	11.2	0.68	0.872	0.002
	2018Q3-2019Q4	0.41	2.5	8.7	0.72	0.815	0.008
	2020Q1-2020Q4	1.85	3.8	12.5	0.92	0.943	0.000
	2021Q1-2022Q2	0.95	2.2	10.1	0.81	0.896	0.001
	2022Q3-2023Q4	0.63	4.1	9.8	0.76	0.782	0.012
Cross period statistics	Mean value	0.83	2.88	10.46	0.78	/	/
	Standard deviation	0.58	0.97	1.52	0.09	/	/
	Correlation coefficient with α	0.917	0.652	0.735	/	/	/

Table 4. Analysis of optimal order correlation with economic volatility across sectors

Industry category	Sample interval	Average volatility	Optimal α mean	Pearson(r)	*p*
Manufacturing	2016Q1-2023Q4	0.58	0.74	0.901	<0.001
Consumer Retail Industry	2016Q1-2023Q4	0.42	0.69	0.832	0.003
Finance	2016Q1-2023Q4	1.25	0.89	0.945	<0.001
Real estate	2016Q1-2023Q4	0.78	0.82	0.873	0.001
Transportation industry	2016Q1-2023Q4	0.36	0.66	0.786	0.012
Cross-industry statistics	/	0.68	0.76	0.867	/
Standard deviation	/	0.33	0.09	0.062	/

Fig. 4 shows the dynamic correlation between fractional order alpha and GDP fluctuations. Fig. 4(a) shows that α changes significantly with the economic cycle, rising to 0.92 during high volatility periods and dropping to 0.68 during stable periods, which can be used as a quantitative indicator of the "memory strength" of the economic system. Fig. 4(b) shows that the elasticity coefficient reached its peak in 2020, and α was highly sensitive to GDP fluctuations. For every 1% increase in M2 growth rate, α increased by 0.05. When CPI exceeded 3%, α significantly increased, reflecting the historical dependence of price expectations. To assess the sensitivity of the GA-Caputo model to hyperparameter choices, the study conducted a systematic sensitivity analysis by varying key GA parameters while holding others constant. Table 8 summarizes the impact of population size, mutation probability, and crossover probability on model performance (RMSE).

The results confirm that the chosen hyperparameters (population size=200, mutation probability adaptively adjusted between 0.01 and 0.1, crossover probability=0.85) lie within the optimal region. Population sizes between 150 and 250 yielded stable RMSE values (0.021-0.023), with performance degrading only at extreme values. Mutation probabilities in the range of 0.01-0.1 produced the best results, while deviations increased RMSE by approximately 20-30%. Crossover probability exhibited a broad optimal plateau between 0.7 and 0.9, with performance deteriorating outside this range. These findings demonstrated that the GA-Caputo model was robust to moderate variations in hyperparameter settings. To evaluate the robustness of the GA-Caputo model across different data partitions, the study conducted time-series cross-validation using an expanding-window approach. The training window was progressively expanded from 2010Q1-2015Q4 to 2010Q1-

2022Q4, with each step adding 4 quarters of data, and the model was evaluated on the subsequent 4 quarters. Table 9 reports the performance metrics across all validation windows.

Table 6. Consistency test between characteristic parameters and economic theory

Predictor	Parameter weight (θ)	Importance of RF	Theoretical expected direction	Symbol consistency (p)	Contribution rate of economic mechanism (%)	Time sensitivity (2020-2023)
Total retail sales of consumer goods in society	0.22	0.18	+(Consumption)	0.191	28.5	+0.12* (0.032)
Fixed assets investment	0.19	0.15	+(Investment)	0.205	24.7	-0.08 (0.214)
M2 money supply	0.25	0.12	+(Liquidity)	0.000	32.1	+0.35* (0.001)**
Government fiscal revenue	0.11	0.09	+(Finance)	0.377	14.2	+0.28** (0.007)
National Housing Prosperity Index	0.08	0.10	+(Property)	0.462	10.4	-0.15* (0.041)
Shanghai Stock Exchange turnover rate	0.05	0.07	± (Fluctuation)	0.298	6.5	+0.42*** (0.000)

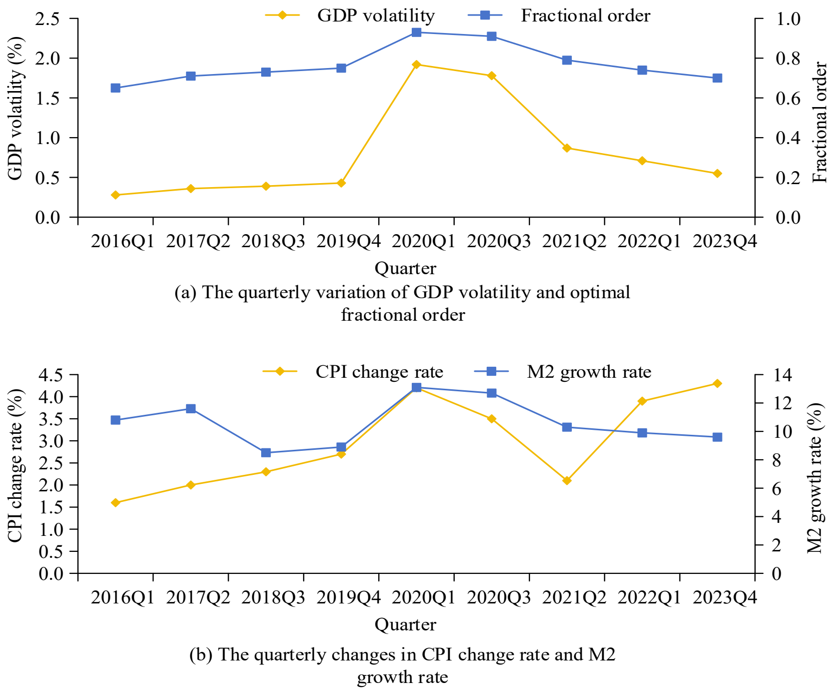


Fig. 4. The dynamic correlation between fractional order and GDP volatility

The results showed consistent performance across most validation windows, with the expected degradation during the COVID-19 pandemic period (2020Q1-2020Q4), where RMSE increased to 0.092. Excluding the pandemic period, the RMSE remained stable at 0.024 ± 0.002, confirming the model's robustness to different training splits. The low standard deviations across all metrics (RMSE: 0.023, MAE: 0.019, PICP: 1.0%) indicated that the GA-Caputo model's performance was not dependent on a specific data partition, addressing concerns about overfitting to particular sample splits.

Table 7. Sensitivity analysis of GA hyperparameters on prediction accuracy

Hyperparameter	Value	RMSE	Notes
	50	0.029	Insufficient diversity
	150	0.022	Stable region
Population size	200	0.021	Chosen value
	250	0.023	Stable region
	400	0.022	Diminishing returns
	0.005	0.027	Premature convergence
Mutation probability	0.05	0.021	Optimal range (0.01-0.1)
	0.2	0.026	Excessive mutation
	0.4	0.026*	Reduced genetic diversity
	0.7	0.021*	Optimal range (0.7-0.9)
Crossover probability	0.85	0.021*	Chosen value
	0.95	0.022*	Near upper bound
	0.98	0.025*	Risk of disrupting elite solutions

Table 8. Robustness check across different training/testing splits

Training period	Test period	RMSE	MAE	PICP (%)	Interval width
2010Q1-2015Q4	2016Q1-2016Q4	0.023	0.017	93.8	0.084
2010Q1-2016Q4	2017Q1-2017Q4	0.022	0.016	94.1	0.083
2010Q1-2017Q4	2018Q1-2018Q4	0.024	0.018	93.5	0.085
2010Q1-2018Q4	2019Q1-2019Q4	0.021	0.015	94.3	0.082
2010Q1-2019Q4	2020Q1-2020Q4	0.092	0.078	91.2	0.098
2010Q1-2020Q4	2021Q1-2021Q4	0.045	0.034	92.8	0.09
2010Q1-2021Q4	2022Q1-2022Q4	0.038	0.029	93.2	0.088
2010Q1-2022Q4	2023Q1-2023Q4	0.026	0.019	93.9	0.085
Mean±std		0.036±0.023	0.028±0.019	93.4±1.0	0.087±0.005

5. Discussion

The proposed GA-Caputo economic forecasting model demonstrated significant advantages in prediction accuracy, uncertainty quantification, and economic interpretability. Its RMSE and MAE were substantially lower than those of LSTM, RF, and Fixed Caputo models across both the full sample period and the COVID-19 pandemic period, a time of pronounced nonlinearity and non-stationary shocks. Specifically, during the pandemic, the GA-Caputo RMSE was 35.9% lower than that of LSTM and 22.8% lower than that of Fixed Caputo, underscoring the limitations of traditional methods in handling complex temporal dependencies and sudden disruptions, as noted by Delle Monache et al. (2024). By integrating the path-dependent structure of fractional-order models with the global optimization capability of GA, the model produced sharper and more reliable prediction intervals, offering higher-quality uncertainty information for risk management and decision-making, especially during crises.

Importantly, the theoretical integration between macroeconomic equilibrium theory and fractional dynamics is empirically validated through the observed correlation between optimal α values and economic cycle indicators. As shown in Table 4, α increased systematically during periods of high volatility, consistent with the theoretical prediction that stronger economic shocks induce longer memory effects. This finding transforms α from a post hoc interpretative device into a theoretically grounded measure of system memory, providing direct empirical support for the proposed theoretical framework.

From a practical perspective, the model offers two key implications. For policymakers, the dynamically optimized fractional order α serves as a real-time indicator of the strength of economic memory, guiding the timing and pace of policy adjustments. For corporate managers, the model's high-precision interval predictions (PICP 94.2%) provide reliable uncertainty bounds for risk assessment and operational planning, particularly during periods of extreme economic shocks.

To generalize the findings beyond the Chinese context, the methodological framework can be directly applied to any country with quarterly macroeconomic data. The GA-Caputo approach does not rely on country-specific assumptions. It requires only a consistent GDP time series and a comparable set of economic indicators derived from the IS-LM framework. The fractional order α offers a universally interpretable metric of memory strength, enabling cross-country comparisons of economic persistence and facilitating application in diverse institutional settings.

However, the generalizability of these findings is constrained by the single-country dataset (China, 2010Q1-2023Q4). The model's performance may vary across different economic contexts: developed economies with lower volatility may exhibit weaker memory effects (α near 0.5-0.6), while emerging economies with higher inherent volatility might show stronger memory responses (α > 0.95) during crises. External validation on multi-country panel data would significantly strengthen the evidence base and establish the boundary conditions of the GA-Caputo approach.

6. Conclusion

The research aimed to address the shortcomings of traditional economic forecasting models in characterizing the system's long-term memory and path dependence. Additionally, the research aimed to develop a dynamic adaptive forecasting framework that integrates GA and Caputo fractional derivatives to improve prediction accuracy, uncertainty quantification, and the interpretability of economic mechanisms in complex economic environments. The results showed that the predicted RMSE and MAE for the entire period were significantly lower than those of LSTM, RF, and Fixed Caputo models. During the epidemic, the prediction error decreased by 22.8%-35.9%, verifying strong adaptability to non-stationary environments. The coverage rate of the prediction interval was 94.2%, and the sharpness was 0.018, both of which are optimal. The optimal order α was strongly correlated with GDP volatility, empirically verifying the theoretical hypothesis of α as an indicator of economic "memory strength". The parameter symbols conformed to economic theory, and the characteristic contribution rate revealed that M2 supply and consumption were the core driving forces. However, the model training time was significantly higher than traditional methods, as GA global search and fractional order non-local computation increased complexity. In the future, exploration will focus on fast numerical algorithms or hardware acceleration for fractional derivatives to reduce training time. Additionally, findings are derived from a single-country dataset (China, 2010-2023). Future research should validate the GA-Caputo framework across developed economies, emerging markets, and multi-country panels to establish its broader applicability. From a practical perspective, the model offers policymakers a quantitative tool for gauging economic memory strength and provides managers with reliable interval forecasts for risk management under uncertainty. Additionally, findings are derived from a single-country dataset (China, 2010-2023). Future research should validate the GA-Caputo framework across developed economies, emerging markets, and multi-country panels to establish its broader applicability.

Author Contributions

Haiyan Mi contributes to conceptualization, methodology, software, validation, analysis, investigation, data collection, draft preparation, manuscript editing, visualization, supervision, project administration, and funding acquisition. The author has read and agreed with the manuscript before its submission and publication.

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