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Abstract: The purpose of this paper is to present an optimization model for planning the distribution of materials in earthmoving operations, considering possible interferences between cut-and-fill sections such as rivers, vegetation, topographical features, or expropriations. The earth allocation problem incorporating interferences was modeled as a linear programming problem, aiming to minimize the total earthmoving cost while considering the constraints related to volume balance, construction project duration, and time for the release of traffic. The proposed linear programming model was run by an integrated system, using Excel for data analysis and IBM CPLEX as the optimizer. The mathematical model was evaluated by a sensitivity analysis and validated by a real-world project of a dam access road in the state of Ceará, Brazil. The unit costs and productivity rates used in the fictional example and in the real-world application followed the referential cost system created by Ceará’s Secretariat of Infrastructure (SEINFRA-CE). The proposed optimization model achieved reasonable processing times for all tested applications, presenting itself as a viable and efficient option for planning earthmoving operations. Furthermore, the linear programming approach provided a 2.12% cost reduction for the real-world case study, when comparing the optimized solution and original budget. This study explored the problem of earth allocation with interferences using a linear programming approach, while avoiding complex modeling issues found in recent literature. As a result, this paper proposes a user-friendly optimization system that can be easily utilized by construction companies and departments.

Keywords: Highway engineering, earthworks, optimization, mathematical programming.

1. Introduction

Construction site logistics need to be financially and environmentally viable in the context of scarce resources (Sidawi, 2012; Whitlock et al., 2018). Consequently, several authors have proposed rational approaches for material allocation to achieve a reduction in construction costs or project duration (Falcão et al., 2016). For example, Burdett and Kozan (2014), Li et al. (2015), Yi and Lu (2016), and Morais and Falcão (2019) utilized operations research techniques to optimize earthworks allocations, equipment fleet scheduling, and equipment routing.

Linear programming (LP) and mixed integer linear programming (MILP) are widely used in earthwork volume allocation as an extension of the classical transportation problem. In general, cut sections and borrow pits are considered as a source of material, while fill sections and disposal sites are considered the material’s final destination. Consequently, mathematical models based on the transportation problem use the volume moved from cut sections/borrow pits to fill sections/disposal sites as a decision variable, presenting a minimization objective related to distances between origin and destination or related to total cost of operations. In addition, most allocation models consider the total cost minimization as an objective function, using unit costs associated with volume in $/m³ (Falcão et al., 2016).

Since the 1980s, authors like Mayer and Stark (1981), Nandgaokar (1981), Easa (1987), Christian and Caldera (1988), Easa (1988), Jayawardane and Harris (1990), Jayawardane and Price (1994a), Güden and Süral (2017), and Gwak et al. (2018) have formulated earth allocation models that do not consider natural or artificial obstacles between cut-and-fill sections such as rivers, vegetation, topographical features, electrical networks, or even expropriations. In other words, all of the formulated models
considered free movement for all possible cut-and-fill combinations, in contrast to real-life construction situations, which typically include several interferences along the transport route.

However, any volume allocation blocked by an interference is discontinued for a specific time that is shorter than the total project duration. For example, if a cut-and-fill combination is blocked by a river, it becomes available after the completion of the construction of a bridge. As a result, this study proposes a linear programming optimization model for planning the distribution of materials in earthworks, aiming to minimize the total cost, incorporating inputs such as obstacle position, equipment fleet production, and duration of obstacle removal (i.e., bridge or culvert construction). In addition, this paper validates the developed model using two case studies, one of them being a real-world road project.

2. Literature Review

Optimization problems related to earthmoving operations can be divided into two sub-categories: equipment fleet planning (EFP) and earth allocation planning (EAP) (Gwak et al., 2018). EFP models have the objective of identifying the best equipment combination, considering fleet productivity and the most advantageous equipment type for distinct excavation scenarios. Some EFP models are incorporated into multifaceted decision support systems (DSS). As an illustration of this, Moselhi and Alshibani (2007) developed an EFP integrated system for real-time control of earthmoving operations. Based on the project data, their model selected near-optimum crew formations by employing a genetic algorithm and monitored the crew productivity through a global positioning system (GPS) tracking module. Thus, it was possible to suggest corrective actions and recalculate a new optimum equipment fleet at any given time during construction. Another example of EFP modeling is shown in Moselhi and Alshibani’s (2009) study, which combined genetic algorithms and an LP model to produce a near-optimum crew plus a least-cost operational plan.

In contrast to EFP models, the EAP formulation is focused on the road project planning phase. EAP models identify the best cut-and-fill combination, considering external sources of earthworks volume (borrow pits) and areas for disposal of excess material from cut sections (disposal sites). Stark and Nicholls (1972) were among the first researchers to suggest an EAP model for planning earthmoving logistics. They argued that LP tools could be even more time-efficient than classical mass diagrams, resulting in solutions that are more economical.

Subsequently, Mayer and Stark (1981) proposed an LP model for minimizing earthworks allocations costs, including three cost categories: excavation and loading, haul, and placement and compaction (embankment). Their LP model was based on volume balance, taking borrow pits, disposal sites, and shrinkage/swell coefficients into consideration. Mayer and Stark (1981) also presented an MILP model as an extension, incorporating set up costs related to borrow pits and disposal sites (i.e., clearing, grubbing, construction or maintenance of access roads, refurbishing, and clean up). In that MILP extension, Mayer and Stark (1981) created a Boolean variable related to set up costs. This variable is conditioned to change depending on whether the borrow pit or disposal site is being used. Consequently, if a borrow pit or disposal site is included in allocations, the Boolean variable would assign a value of 1; otherwise, it would assign a value of 0.

Mayer and Stark (1981) paved the way for more advanced EAP models that considered other engineering complexities. As a result, subsequent research attempted to add constraints related to specific road construction characteristics. Between the late 1980s and the 1990s, five studies by the following researchers used the formulation by Mayer and Stark as a basis: Easa (1987), Easa (1988), Jayawardane and Harris (1990), Jayawardane and Price (1994a), and Jayawardane and Price (1994b).

Easa (1987) and Easa (1988) discussed the relevance of using non-constant costs. Although the unit cost for haul varies depending on the distance travelled between the source and destination, other cost components in previous models were equal for any quantity of excavated volume. Therefore, Easa (1987) added constraints to the model by Mayer and Stark (1981), considering volume-dependent unit costs. These costs were set to change depending on the volume interval. For example, the unit cost for a borrow pit-fill section allocation would be 8.20 USD/m³ for an excavated volume between 0 m³ and 1000 m³ and 7.30 USD/m³ for an excavated volume between 1000 m³ and 2000 m³.

In contrast, Easa (1988) proposed a linear function for the borrow pit-fill section and cut section-disposal site allocation cost. This function used the excavated volume as the main variable. Consequently, the optimization model based on the work by Mayer and Stark (1981) became a quadratic programming (QP) model. Thus, Easa (1988) explored solution methods to find a global minimum taking into consideration the nature of the quadratic programming. However, he concluded that the global minimum was guaranteed only when unit cost functions were non-decreasing.

Jayawardane and Harris (1990) created an innovative approach to EAP modeling, formulating an additional constraint related to the total duration of the project costs (Fernandes and Espindola, 2019). This constraint consists of the ratio of earth volume allocated (m³) to equipment fleet productivity (m³/day), which must be less than or equal to the total project duration. As an extension, Jayawardane and Price (1994a) developed an integrated system using Jayawardane and Harris’s (1990) MILP model combined with a computer simulation to estimate productivity indexes and unit costs. Subsequently, Jayawardane and Price (1994b) presented two numerical examples, outlining their simulation-optimization system operation and performance. Although this integrated system provided satisfactory estimations of costs and productivity, it is necessary to run several simulations for each equipment team and for each feasible operation between cut-and-fill sections, borrow pit and fill sections, and cut sections and disposal pits. Therefore, this approach may be limited when there is no consistent database. Additionally, it can be too complex or time-consuming in large-scale highway projects.

In the last two decades, some LP and MILP approaches have stood out in EAP research. Ji et al. (2010), Hare et al. (2011), Lima et al. (2013), Yi and Lu (2016), and Choudhari and Tindwani (2017) presented different views on road construction planning. For example, Ji et al. (2010) considered non-cooperating construction companies working in the same road project. Consequently, they
formulated an MILP approach for planning earthwork section division, working on a previous solution obtained by an LP model for allocation. In contrast, Yi and Lu (2016) developed a MILP model for temporary haul road design, while Lima et al. (2013) and Choudhari and Tindwani (2017) expanded allocation models for paving operations. Lima et al. (2013) inserted Boolean variables to decide the cheapest way to mix material for soil-aggregate pavement layers. In that case, the model had to choose between in situ mixing or mixing in manufacturing plants. The LP approach used by Choudhari and Tindwani (2017) also considered the allocation of mixed material from different sources to the pavement layer, using an intermediary destination for processing material.

Hare et al. (2011) held a distinct view on EAP modeling, questioning highway continuity during construction. They observed that natural interferences or physical blocks could significantly change earthmoving operations and logistics. For instance, trucks cannot transport earth from a cut section to a fill section separated by a river. As a consequence, Hare et al. (2011) created an MILP model for earth allocation, including time steps as a new coordinate in volume decision variables. Therefore, the MILP decision variable represents the earth moved from cut \( i \) (or borrow pit \( i \)) to section \( j \) (or waste pit \( j \)) during time step \( t \). In addition, the authors introduced a binary variable to indicate whether the block was removed during a certain time step.

After formulation, Hare et al. (2011) verified their MILP model’s processing performance. As a result, they concluded that the computation time for a significant number of road sections is significantly high. Consequently, Hare et al. (2011) introduced a set of algorithms for solving time reduction, creating heuristics for finding a feasible starting point to repeat MILP solving.

In contrast to Hare et al. (2011), this paper brought a novel approach that aims to solve the EAP with interferences using a solid optimization methodology integrated to a LP model, which is not computationally expensive. Thus, this paper had the objective of utilizing fewer resources such as time, memory and processing capacity while proposing an accessible tool for managers. The LP model proposed integrated time management constraints based on estimated productivity of equipment fleet and project deadlines. Therefore, the proposed approach was developed for being both a mathematical model for optimizing earthwork allocations and a tool for scheduling road projects.

### 3. Proposed Approach

#### 3.1. Scope

Before formulation, it was necessary to set the EAP model characteristics and requirements. The points are listed below:

- The mathematical programming model has to minimize the total earthwork cost;
- It must use reasonable and standardized unit costs as well as productivity rates;
- It must consider constraints related to total project duration;
- It needs to block some cut-and-fill allocations while interferences are being removed;
- The model needs to provide satisfactory computational performance, avoiding the use of complex algorithms that deliver near-optimum solutions.

#### 3.2. Remarks on Unit Cost and Productivity

In contrast to Easa (1987), Easa (1988), and Jayawardane and Price (1994a), this study considered unit costs and productivity rates based on a referential cost system created by Ceará’s Secretariat of Infrastructure (SEINFRA-CE) (SEINFRA-CE, 2019). This system calculates productivity rates and costs based on real-construction observations, incorporating inputs such as soil type, distances, necessary equipment, manpower, efficiency factors, and productive and unproductive times. Although this approach can lead to a conservative productivity estimation, it is a reasonable way to standardize productivity rates for a specific region. Consequently, it is not necessary to run an exhaustive number of computer simulations, once productivity is calculated using a statistically based approach.

#### 3.3. Modeling

The model formulation assumed \( S \) as the set of all cut sections, \( CA \) as the set of all fill sections, \( J \) as the set of all borrow pits, \( K \) as the set of all disposal sites and \( W \) as the set of all steps for equipment traffic release, considering that one block is removed by step after the step 1 (initial condition). Thus, it was presumed that all cut section \( s \in S \) and all fill section \( ca \in CA \) for each step \( t \), where \( t \in W \), having one block removed for each \( t > 1 \). As a consequence, a new set \( C \) had to be created to represent the blocked cut-fill allocations:

\[
C = \{(s, ca) | s \text{ and } ca \text{ are located on opposite sides of block } b\}
\]

Two vectors \((T(t) \text{ and } TK(b))\) was also introduced as inputs that respectively represent the actual time of construction and the time of block \( b \) removal. Therefore, for all \((s, ca) \in C\) and for time \( T(t) < TK(b) \):

\[
XS (s, ca, t) = 0
\]

where \( XS \) ("in situ" volume in \( m^3 \)) is the decision variable that corresponds to the volume of material allocated from cut section \( s \) to fill section \( ca \) in step \( t \). Similarly, the model assumed borrow pit \( i \in J \), considering another set for borrow pits \( i \) in which its access road is located on the opposite side of a fill section \( ca \).

\[
CJ = \{(i, ca) | \text{borrow pit’s } i \text{ access road and } ca \text{ are located on opposite sides of block } b\}
\]

Therefore, for \((i, ca) \in CJ \text{ and } T(t) < TK(b) \):

\[
XB (i, ca, t) = 0
\]

where \(XB\) ("in situ" volume in \( m^3 \)) is the decision variable that corresponds to the volume of material allocated from borrow pit \( i \) to fill section \( ca \) in time \( t \).

It was assumed that excess material from cut sections will be discarded at the closest disposal sites (i.e., roadside spoil pits) that meet all environmental requirements. Consequently, the model included another set for disposal
site \( k \in K \), whose access road is located on the opposite side of a cut section \( s \): \[
CK = \{(k,s)\mid \text{disposal site's k access road and s are located on opposite sides of block b}\}
\] (5)

Therefore, for \((k, s) \in CK\) and \(T(t) < TK(b)\), \[
XD (k, s, t) = 0
\] (6)

where \( XD \) (“in situ” volume in \( \text{m}^3 \)) is the decision variable that corresponds to the volume of material allocated from cut section \( s \) to disposal site \( k \) in time \( t \).

After formulating constraints related to blocked allocations in equations Eq. (1) to Eq. (6), the time constraint for cut-disposal allocations was considered in the proposed model, considering a new set \( WT \) that incorporated the steps in which operation’s time \((T(t))\) corresponds to block removal time \((TK(b))\):
\[
WT = \{t \mid T(t) = TK(b)\}
\] (7)

Thus, for each block \( b \) and for all \( s \in S, ca \in CA \), and \( t \in WT \),
\[
\sum_{s \in S} \sum_{ca \in CA} \sum_{t \in WT} XS(s, ca, t) \left(\frac{1}{PS}\right) + TK(b) \leq D
\] (8)

where \( PS \) is the equipment fleet production (in \( \text{m}^3/\text{day} \)) in cut-fill allocations and \( D \) is total project duration in days. Likewise, the time constraint for the borrow-pit allocations was also included. In summary, for all \( i \in J \) and \( ca \in CA \),
\[
\sum_{i \in J} \sum_{ca \in CA} \sum_{t \in WT} XB(i, ca, t) \left(\frac{1}{PB}\right) + TK(b) \leq D
\] (9)

where \( PB \) is the equipment fleet production (in \( \text{m}^3/\text{day} \)) in borrow-pit allocations. Similarly, the time constraint for cut-disposal site allocations was considered in the formulation. Thus, for all \( s \in S \) and \( k \in K \),
\[
\sum_{s \in S} \sum_{k \in K} \sum_{t \in WT} XD(k, s, t) \left(\frac{1}{PD}\right) + TK(b) \leq D
\] (10)

where \( PD \) is the equipment fleet production (in \( \text{m}^3/\text{day} \)) in cut-disposal site allocations and \( XD \) (“in situ” volume in \( \text{m}^3 \)) is the decision variable that corresponds to the volume of material allocated from cut section \( s \) to disposal site \( k \) in time \( t \).

Schedule is another important consideration. It was supposed that the equipment performs operations related to \( PS, PB \), and \( PD \), working in parallel. As a result, it was not necessary to add a fourth coordinate related to the equipment team.

In order to complete the earth allocation LP model, the objective function \( Z \) was formulated as
\[
\min Z = \sum_{s \in S} \sum_{ca \in CA} \sum_{t \in WT} CS(s, ca) \times XS(s, ca, t) + \sum_{k \in K} \sum_{s \in S} \sum_{t \in WT} CD(k, s) \times XD(k, s, t) + \sum_{i \in J} \sum_{ca \in CA} \sum_{t \in WT} CB(i, ca) \times XB(i, ca, t)
\] (11)

The objective function included all costs associated with earth allocation, aiming to minimize total project costs. Henceforth, the volume balance constraints were added to this LP formulation, considering:

- For all cut section \( s \):
\[
\sum_{ca \in CA} \sum_{t \in WT} XS(s, ca, t) + \sum_{k \in K} \sum_{t \in WT} XD(k, s, t) = VC(s)
\] (12)

- For all borrow pit \( i \):
\[
\sum_{ca \in CA} \sum_{t \in WT} XB(i, ca, t) \leq VOL(i)
\] (13)

- For all fill sections, \( ca \):
\[
\sum_{s \in S} \sum_{t \in WT} XS(s, ca, t) \times FS(s, ca) + \sum_{i \in J} XB(i, ca, t) \times FI(i, ca) = VA(ca)
\] (14)

- For each disposal site, \( k \):
\[
\sum_{s \in S} \sum_{t \in WT} XD(k, s, t) \times FKCA(k, s) \leq VBF(k)
\] (15)

The following notation was used in the objective function and volume balance constraints:

- \( CB(i, ca) \) = Total unit cost in USD/\( \text{m}^3 \) to allocate material from borrow pit \( i \) to fill section \( ca \) (“in situ” volume);
- \( CD(k, s) \) = Total unit cost in USD/\( \text{m}^3 \) to allocate material from cut section \( s \) to disposal site \( k \) (“in situ” volume);
- \( CS(s, ca) \) = Total unit cost in USD/\( \text{m}^3 \) to allocate material from cut section \( s \) to fill section \( ca \) (“in situ” volume);
- \( FI(i, ca) \) = Bulking/swell factors of material from borrow pit \( i \) that will be placed and compacted in fill section \( ca \);
- \( FKCA(k, s) \) = Bulking/swell factors of material from cut section \( s \) that will be placed and compacted in the fill section \( ca \);
- \( VA(ca) \) = Volume of fill section \( ca \), in \( \text{m}^3 \);
- \( VBF(k) \) = Volume of disposal pit \( k \), in \( \text{m}^3 \);
- \( VC(s) \) = Volume of cut section \( s \), in \( \text{m}^3 \); and
- \( VOL(i) \) = Volume of borrow pit \( i \), in \( \text{m}^3 \)
3.4. Optimization Methodology

Before utilizing the proposed LP model (Equations Eq. (1) to Eq. (15)), a five-item protocol was developed in Excel to analyze road projects and obtain model inputs. First, the volumes and geotechnical characteristics of each section were obtained by examining the geometric project and soil surveys. Second, interference positions and their removal time were provided. The third and fourth steps are related to unit costs and productivity rate calculation, using geotechnical data and distances between sections for estimating equipment performance in operations such as excavation, loading, hauling, placement, and compaction. Finally, the total project duration was set.

After insertion of the input, a few steps remain in order to achieve optimal distributions, as shown in Fig. 1.

![Fig. 1. Steps for model resolution: (A) Material properties, geometric and geotechnical data; (B) Interferences data: positioning and release times; (C) Unit costs; (D) Equipment fleet productivity; (E) Total project duration](image)

Initially, the Excel spreadsheet computes all unit costs and productivity rates for all possible earth allocations, calculating the cost and productivity associated with $CB(i, ca)$, $CD(k, s)$, and $CS(s, ca)$. The next step is related to the LP model’s implementation and solution where IBM CPLEX version 12.6.0.0 was used, employing the Integrated Development Environment (IDE) and Optimization Programming Language (OPL) (IBM, 2012). Consequently, it is possible to link Excel and IBM CPLEX, obtaining and pre-processing inputs such as calculated cost arrays, productivity rates, time vectors, project time duration, and bulking/swell factors. Finally, the following results are presented by CPLEX using post-processing resources (ILOG Script):

- movement of earthwork volume from cuts to fill sections (quantity, origin, and destination);
- movement of earthwork volume from borrow pits to fill sections (quantity, origin, and destination);
- movement of earthwork volume from cut sections to disposal sites (quantity, origin, and destination); and
- time steps for each movement, considering time for the removal of blocks and free equipment traffic.

According to the aforementioned results, schedules are constructed for variables $XB(i, ca, t)$, $XD(k, s, t)$, and $XS(s, ca, t)$.

4. Numerical Examples

This paper used two examples for validation and analysis, possibly leading the applicability of the model to a limited range of tests. However, it was possible to evaluate the main parameters and investigate the performance of the proposed model through the analysis of different scenarios, as shown in Example 1. Furthermore, Example 2 evaluated how the proposed LP model can be applied in practice, showing the possible effects related to planning and scheduling in real earthmoving operations.

All numerical applications were run in IBM CPLEX version 12.6.0.0 on a PC with an Intel Core i5 2.3 GHz processor and 4 GB of memory.

4.1. Example 1

This example had the objective of evaluating how earthmoving allocations can be planned incorporating interferences in LP modeling. Consequently, this example presents a sensitivity analysis that evaluates how the number of blocks considered and different deadlines (D) can influence total construction costs. In Example 1, (Fig. 2), a 36 km long road was built with four blocks, six borrow pits, three disposal areas, twenty-four cut sections, and fifty-one fill sections, presenting volumes, costs and results detailed in Lima et al. (2020) published data.

![Fig. 2. Existing ground profile and road vertical profile](image)
Firstly, Example 1 was run with several project periods ($D$) in order to observe two main aspects: the time in $D$ that starts to generate unfeasible results, and the time in $D$ where the costs start to be constant, as shown in Fig. 3.

As a result, Fig. 3 shows that $D = 103$ days is the shortest feasible duration, and also demonstrates that after $D = 123$ days, the project total cost will not change. Thus, in the $D > 123$ days scenario, it was possible to infer that LP model was able to wait for blocks removal, allowing cheaper allocations between supply (cut sections and borrow pits) and demand (fill sections and disposal areas). In contrast to $D > 123$ days, the scenarios with shorter deadlines did not have full availability of some blocked allocations, forcing the LP model to choose costly alternatives. As an illustration of that, the 103-day- scenario was 11.54% more expensive than the 123-day- scenario.

After analyzing project deadlines influence on costs, Example 1 evaluated the importance of interferences in modeling, comparing the difference in costs considering the blocks or not. Therefore, it was considered a constant project duration $D = 103$ days and five scenarios (Fig. 4) where the first one ignored the four blocks and considered the 36 Km road as a continuous segment while the other scenarios consecutively included one to four blocks, following the sequence of interferences showed in Fig. 2.

As shown in Fig. 4, the earthwork costs are proportional to the number of blocks considered, presenting a 13.05% of the cost increase when comparing the scenario with no blocks and the scenario with four blocks. Consequently, it was possible to observe that ignoring blocks also results in ignoring additional costs related to time management constraints, changing the hauling plans. As an example (Fig. 5), the scenario with three blocks did not consider Block 4 in contrast to the last scenario with all blocks. As a result, the fill section 44 to 51 received 17,333 m$^3$ of earth from cut sections 21 to 24 and borrow pits J1, J2, J4 and J5, and just 2,889 m$^3$ from J6, which is the most expensive borrow pit. However, if Block 4 is considered, fill sections 44 to 51 can only receive material from cut sections and J1, J2, J4 and J5 borrow pits after 90 days, limiting the time window for allocations of “non-J6” material. Therefore, the fill sections 44 to 51 would receive 4000 m$^3$ from J6 and 15,423 m$^3$ from other sources (J1, J2, J4, and J5, and cut sections 9 and 12) in the 4 block-scenario, representing USD 37,597.14 in additional costs just for filling the segment between fill sections 44 to 51.

During the analysis of the blocks influence on costs, it was also noticed that the smallest cost change was between the two-blocks (Block 1 and 2) and one-block scenarios (just Block 1), it is related to two main aspects. First, Block 2 has the shortest time for block removal. Consequently, there are few allocations blocked by Block 2 that are more advantageous than waiting for its removal in 60 days, generating just small changes in the total costs between one-block (without Block 2) and two-blocks scenarios (with Block 2). Second, Block 2 has a just a small influence in allocations near Block 1. Thus, few of these allocations changed when Block 2 was considered (two-blocks-scenario) or not (one-block-scenario).
The objective of Example 2 was to make the proposed model applicable in real practice. As a result, the LP model was used in real road project, which has a considerable number of interferences. Additionally, this example also compared the optimized and original project budget in order to evaluate if this approach can lead to a significant difference on costs.

This example was based on the Maranguapinho dam maintenance road located between the municipalities of Maracanaú and Maranguape in the state of Ceará, Brazil. This road was designed to provide access to Maranguapinho’s weir and power transmission lines, being 10.58 km long and 5.30 m wide. This access road has a closed-circuit horizontal alignment (Fig. 6), where the equipment fleet can only access it at four points: the road start and end, and borrow pit accesses (J1 and J2). Consequently, equipment teams cannot work on the roadsides. Thus, road alignment and borrow pit accesses are the only ways to haul the material. Moreover, six interferences were included in this project. All of them are related to culvert construction, as shown in Fig. 6.

Before running Example 2, the cut and fill sections volumes were defined based on the geometric project and geotechnical studies. As a consequence, it was found that Maranguapinho dam’s maintenance road was divided in five cut sections and fifty-two fill sections, as shown in Table 1.

<table>
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<th>Cut</th>
<th>Volume (1000 m³)</th>
<th>Fill</th>
<th>Volume (1000 m³)</th>
<th>Fill</th>
<th>Volume (1000 m³)</th>
<th>Fill</th>
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<td>0.260</td>
<td>46</td>
<td>0.464</td>
<td>47</td>
<td>0.452</td>
</tr>
<tr>
<td>8</td>
<td>0.567</td>
<td>21</td>
<td>2.523</td>
<td>34</td>
<td>0.627</td>
<td>48</td>
<td>0.595</td>
<td>49</td>
<td>0.981</td>
</tr>
<tr>
<td>9</td>
<td>0.524</td>
<td>22</td>
<td>3.336</td>
<td>35</td>
<td>0.674</td>
<td>48</td>
<td>0.595</td>
<td>50</td>
<td>1.426</td>
</tr>
<tr>
<td>10</td>
<td>1.037</td>
<td>23</td>
<td>3.650</td>
<td>36</td>
<td>0.480</td>
<td>49</td>
<td>0.981</td>
<td>51</td>
<td>0.752</td>
</tr>
<tr>
<td>11</td>
<td>0.359</td>
<td>24</td>
<td>3.071</td>
<td>37</td>
<td>0.619</td>
<td>50</td>
<td>1.426</td>
<td>52</td>
<td>0.441</td>
</tr>
<tr>
<td>12</td>
<td>0.986</td>
<td>25</td>
<td>2.491</td>
<td>38</td>
<td>0.551</td>
<td>51</td>
<td>0.752</td>
<td>52</td>
<td>0.441</td>
</tr>
<tr>
<td>13</td>
<td>2.703</td>
<td>26</td>
<td>0.379</td>
<td>39</td>
<td>0.547</td>
<td>52</td>
<td>0.441</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 5. Allocations without and with Block 4

4.2. Example 2

The objective of Example 2 was to make the proposed model applicable in real practice. As a result, the LP model was used in real road project, which has a considerable number of interferences. Additionally, this example also compared the optimized and original project budget in order to evaluate if this approach can lead to a significant difference on costs.

This example was based on the Maranguapinho dam maintenance road located between the municipalities of Maracanaú and Maranguape in the state of Ceará, Brazil. This road was designed to provide access to Maranguapinho’s weir and power transmission lines, being 10.58 km long and 5.30 m wide.
Following the optimization methodology (Section 3.4), the unit costs were calculated as well as other necessary information was imported to IBM CPLEX such as the block position and release times, fleet productivity indexes, and project duration (D=100 days), being detailed in Lima et al. (2020) data. As a result, the LP model presented a total cost ($Z = USD 179,409.47$) and the optimized allocated volumes through the decisions variables $XS$ for cut-fill allocations (Table 2), $XB$ for borrow pit-fill allocations (Table 3), and $XD$ for cut-disposal area allocations. However, the only disposal area was not used once all material of cut sections was transported to fill pits. Thus, all $XD$ values were assumed to be numerically null.

Table 2 presents all optimized volumes related to cut-fill allocations ($XS$), describing the origin of each allocation $s$, destination $ca$, and start time $T(t)$.

**Table 2. Volumes of the allocations between cut sections and fill sections ($XS (s, ca, t)$)**

<table>
<thead>
<tr>
<th>$s$ (Cut section)</th>
<th>ca (Fill section)</th>
<th>$T(t)$ (Days start)</th>
<th>Volume (1000 m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>31</td>
<td>60</td>
<td>0.065</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>60</td>
<td>0.378</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>45</td>
<td>0.008</td>
</tr>
<tr>
<td>4</td>
<td>29</td>
<td>45</td>
<td>0.136</td>
</tr>
<tr>
<td>5</td>
<td>33</td>
<td>80</td>
<td>0.067</td>
</tr>
</tbody>
</table>

On the other hand, Table 3 presents all optimized volumes related to borrow pit-fill allocations ($XB$), describing the origin of each allocation $i$, destination $ca$, and start time $T(t)$.

**Table 3. Volumes of the allocations between borrow pits and fill sections ($XB (i, ca, t)$)**

<table>
<thead>
<tr>
<th>$i$ (Borrow pit)</th>
<th>ca (Fill section)</th>
<th>$T(t)$ (Days start)</th>
<th>Volume (1000 m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>30</td>
<td>0.540</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>45</td>
<td>1.128</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>45</td>
<td>0.744</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>45</td>
<td>0.276</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>45</td>
<td>0.440</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>45</td>
<td>1.192</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>45</td>
<td>0.562</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>45</td>
<td>1.060</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>45</td>
<td>0.413</td>
</tr>
<tr>
<td>10</td>
<td>11</td>
<td>45</td>
<td>1.125</td>
</tr>
<tr>
<td>11</td>
<td>12</td>
<td>45</td>
<td>3.107</td>
</tr>
<tr>
<td>12</td>
<td>13</td>
<td>45</td>
<td>3.488</td>
</tr>
<tr>
<td>13</td>
<td>14</td>
<td>60</td>
<td>1.562</td>
</tr>
<tr>
<td>14</td>
<td>15</td>
<td>60</td>
<td>1.689</td>
</tr>
<tr>
<td>15</td>
<td>16</td>
<td>60</td>
<td>1.118</td>
</tr>
<tr>
<td>16</td>
<td>17</td>
<td>60</td>
<td>0.554</td>
</tr>
<tr>
<td>17</td>
<td>18</td>
<td>60</td>
<td>0.909</td>
</tr>
<tr>
<td>18</td>
<td>19</td>
<td>60</td>
<td>1.578</td>
</tr>
<tr>
<td>19</td>
<td>20</td>
<td>60</td>
<td>2.900</td>
</tr>
<tr>
<td>20</td>
<td>21</td>
<td>60</td>
<td>3.835</td>
</tr>
<tr>
<td>21</td>
<td>22</td>
<td>60</td>
<td>4.195</td>
</tr>
<tr>
<td>22</td>
<td>23</td>
<td>60</td>
<td>2.485</td>
</tr>
<tr>
<td>23</td>
<td>24</td>
<td>60</td>
<td>0.436</td>
</tr>
</tbody>
</table>

With the times $T(t)$, the volumes $XS$ and $XB$, and the estimated productivities ($PS$, $PB$ and $PD$ in 1000 m³/day) (Lima et al., 2020), it was possible to build a project schedule where x-axis represents time in the construction project and y-axis corresponds to the duration of allocations $XS$ and $XB$ in a specific start time, as shown in Figure 7.

**Fig. 7. Example 2 Schedule**

The schedule in Fig. 7 presents two important characteristics of considerable interferences in LP modeling. First, it shows that the model considered distinct equipment fleet working in parallel where both cut-fill and borrow pit-fill operations were executed simultaneously. Second, it shows that blocks can generate long breaks between the removal of interferences for release of
equipment traffic, generating delays due to the limitation of resources. To illustrate this, there are two breaks in Fig. 7. The first of them is located between the end of borrow pit-fill allocations started at \( T = 0 \) days and the removal of blocks 1 and 4 \((\approx 26.8 \text{ days})\), and the second is located between the end of borrow pit-fill allocations started at \( T = 30 \) days and the removal of block 3 \((\approx 8.8 \text{ days})\).

Finally, a new budget (Table 4) was made based on the optimized results and the costs of SEINFRA-CE (2019) where three types of unit cost for earthmoving operations were used: a unit cost for excavation, loading and hauling based on distance intervals; a unit cost for compaction based on compacted volume; and an additional cost for hauling material for distances greater than 5 Km. Additionally, this table also presents the original budget for comparison.

The optimized budget presented 2.12% cost reduction compared to the original project, which did not consider interference in allocation planning. Consequently, the original solution not only ignored the blocks, but also chose for more expensive allocations. For instance, 79.3% \((47,116.68 \text{ m}^3)\) of the original project allocations have distances greater than 2 km while the optimized solution presented 50.9% \((30,250.62 \text{ m}^3)\), as shown in Table 4.

### 4.3. Model’s Performance

In general, the processing times (including data import) were considerably low for all tested examples and scenarios. In Example 1, the first analysis related to project duration tested 28 scenarios in which the processing time took an average value of 3.49 s, presenting a minimum value of 3.01 s and a maximum value of 3.92 s. For the second analysis, the five scenarios (0 to 4 blocks) presented an average of 3.66 s, a minimum value of 3.45 s and a maximum value of 3.93 s. On the other hand, Example 2 presented a higher processing time value of 4.26 s.

### 5. Conclusion

In summary, this paper proposes an LP approach for an earth allocation problem with physical interferences, using standardized unit costs and productivity rates based on field data (SEINFRA-CE cost system). As a consequence, this LP model did not need to employ integrated simulation-optimization approaches. In other words, it was also possible to present satisfactory and viable solutions without exhaustive discrete-event simulations.

Another contribution of this LP model is related to computational performance, where it was not necessary to apply complex algorithms or metaheuristics, as in previous

---

**Table 4. Optimized and original budget**

<table>
<thead>
<tr>
<th>Item</th>
<th>Unit</th>
<th>Original Budget</th>
<th>Optimized Budget</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Quantity</td>
<td>Cost (USD)</td>
</tr>
<tr>
<td>Excavation, loading and hauling from 50 to 200 m</td>
<td>m³</td>
<td>1.56</td>
<td>8,328.46</td>
</tr>
<tr>
<td>Excavation, loading and hauling from 200 to 400 m</td>
<td>m³</td>
<td>1.73</td>
<td>0.00</td>
</tr>
<tr>
<td>Excavation, loading and hauling from 400 to 600 m</td>
<td>m³</td>
<td>1.82</td>
<td>873.80</td>
</tr>
<tr>
<td>Excavation, loading and hauling from 600 to 800 m</td>
<td>m³</td>
<td>2.01</td>
<td>0.00</td>
</tr>
<tr>
<td>Excavation, loading and hauling from 800 to 1000 m</td>
<td>m³</td>
<td>2.11</td>
<td>0.00</td>
</tr>
<tr>
<td>Excavation, loading and hauling from 1000 to 1200 m</td>
<td>m³</td>
<td>2.14</td>
<td>2,960.43</td>
</tr>
<tr>
<td>Excavation, loading and hauling from 1200 to 1400 m</td>
<td>m³</td>
<td>2.33</td>
<td>0.00</td>
</tr>
<tr>
<td>Excavation, loading and hauling from 1400 to 1600 m</td>
<td>m³</td>
<td>2.41</td>
<td>135.74</td>
</tr>
<tr>
<td>Excavation, loading and hauling from 1600 to 1800 m</td>
<td>m³</td>
<td>2.58</td>
<td>0.00</td>
</tr>
<tr>
<td>Excavation, loading and hauling from 1800 to 2000 m</td>
<td>m³</td>
<td>2.67</td>
<td>0.00</td>
</tr>
<tr>
<td>Excavation, loading and hauling from 2000 to 3000 m</td>
<td>m³</td>
<td>2.74</td>
<td>42,755.08</td>
</tr>
<tr>
<td>Excavation, loading and hauling from 3000 to 4000 m</td>
<td>m³</td>
<td>3.16</td>
<td>0.00</td>
</tr>
<tr>
<td>Excavation, loading and hauling from 4000 to 5000 m</td>
<td>m³</td>
<td>3.50</td>
<td>0.00</td>
</tr>
<tr>
<td>Excavation and loading &gt; 5000 m</td>
<td>m³</td>
<td>1.04</td>
<td>4,361.60</td>
</tr>
<tr>
<td>Hauling* &gt; 5000 m (Distance = 5.08 Km)</td>
<td>ton</td>
<td>0.81</td>
<td>0.00</td>
</tr>
<tr>
<td>Hauling* &gt; 5000 m (Distance = 5.16 Km)</td>
<td>ton</td>
<td>0.82</td>
<td>0.00</td>
</tr>
<tr>
<td>Hauling* &gt; 5000 m (Distance = 5.17 Km)</td>
<td>ton</td>
<td>0.82</td>
<td>0.00</td>
</tr>
<tr>
<td>Hauling* &gt; 5000 m (Distance = 5.28 Km)</td>
<td>ton</td>
<td>0.83</td>
<td>0.00</td>
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<tr>
<td>Hauling* &gt; 5000 m (Distance = 5.48 Km)</td>
<td>ton</td>
<td>0.86</td>
<td>0.00</td>
</tr>
<tr>
<td>Hauling* &gt; 5000 m (Distance = 5.68 Km)</td>
<td>ton</td>
<td>0.88</td>
<td>0.00</td>
</tr>
<tr>
<td>Hauling* &gt; 5000 m (Distance = 5.77 Km)</td>
<td>ton</td>
<td>0.90</td>
<td>7,545.57</td>
</tr>
<tr>
<td>Hauling* &gt; 5000 m (Distance = 5.88 Km)</td>
<td>ton</td>
<td>0.91</td>
<td>0.00</td>
</tr>
<tr>
<td>Hauling* &gt; 5000 m (Distance = 6.08 Km)</td>
<td>ton</td>
<td>0.93</td>
<td>0.00</td>
</tr>
<tr>
<td>Landfills compaction</td>
<td>m³</td>
<td>0.65</td>
<td>51,665.31</td>
</tr>
</tbody>
</table>

Total budget amount | 183,303.62 | 179,409.47 |

*SEINFRA-CE (2019) considers the cost of hauling as a distinct cost for allocations where the distance between origin and destination is longer than 5 Km, following this linear function: cost = distance (in Km) \( \times 0.124 + 0.18 \)
studies. Although several constraints were added, the LP formulation did not utilize binary variables, which can exponentially increase the solving time and the size of the underlying linear program. Therefore, the proposed LP approach can be considered as a non-computationally intensive model. As a result, all model solutions were solved in extremely low time, as shown in Section 4.3.

The examples not only validated the proposed LP model, but also presented technical characteristics. For instance, the analysis presented in examples 1 and 2 showed how the inclusion of interference in modeling could affect costs and scheduling. In general, the inclusion of blocks in Example 1 showed how additional cost related to interferences could be simply ignored in project planning, possibly raising the construction costs with unexpected corrections. Another important point observed in Example 1 was the relationship between total project duration and total earthmoving costs. It was possible to conclude that shorter deadlines will provide costly allocations since there is less time for completion of operations and for block removal.

Among the possible limitations, this article validated the model using just two numerical examples based on a Brazilian cost system. Consequently, the LP model and optimization methodology may need few adaptations for being implemented in other contexts.

Finally, the employed optimization methodology also showed itself as user-friendly since the protocol for obtaining and pre-processing data was developed in Excel. The link between Excel and IBM CPLEX version 12.6.0.0 presented no connectivity issues. Thus, this solution proposal can be easily applied in civil engineering companies or governmental institutions like Federal or State Highway Departments.

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