A MIXED INTEGER PROGRAMMING FOR A VEHICLE ROUTING PROBLEM WITH TIME WINDOWS: A CASE STUDY OF A THAI SEASONING COMPANY

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Abstract

The goal of this study was to improve the transportation management of a case study company which produced seasoning powder. In the past, there were two major problems in logistics department: high transportation cost and long vehicle scheduling time. Thus, the objectives of this study were to reduce transportation cost and scheduling time. Due to the delivery in Bangkok and metropolitan area, this problem became the vehicle routing problem with time windows. Then, we proposed a mixed integer programming to minimize the total cost of fixed and variable cost of vehicles and transportation cost. However, instead of formulating the time interval constraint as a general VRPTW problem, we defined a binary parameter to represent whether the customer has a time window constraint. Then, we used CPLEX to solve the problem. The result showed that after implementing the MIP, the monthly transportation cost was reduced by 23% or 9,413 baht per week and the planning time was reduced by 67%.

Keywords: Vehicle Routing Problem with Time Window, Mixed Integer Programming, Zoning

1. INTRODUCTION

The transportation cost is one of the highest portion of logistics cost in several organizations. The major problem in the transportation department in any industry is vehicle routing problem (VRP). It is to decide the sequence of delivery or pickup points, which may be visited by a delivery vehicle, starting and ending at some depots. Examples of routing problems would be truck dispatching when the demand of customers on the route is less than a truckload (LTL). As the fuel price increases continuously, the effective routing and scheduling can result in tremendous savings by focusing on areas such as increased productivity and better vehicle utilization. In this research, we focused on solving the vehicle routing in Bangkok and metropolitan area where there is the restriction of the delivery time due to rush hour traffic. Hence, the routing problem became more complex because the requirement of delivery time of the particular area before 8 a.m. This research focused on

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solving the problem in logistics department of the case study company which produces seasoning powder and delivers products to customers by third party logistics (3PLs). In 2011, there were 690 customers in Bangkok and metropolitan area using 26 trips per week. The 3PLs distributed the products to customer according the delivery plan made by logistics department. In the past, it solved the routing problem based on the officer's experiences and sent the delivery schedule to the drivers of the 3PLs. However, Ongarj and Ongkunaruk (2013) found the transportation management system was inefficient. The officer spent three hours to schedule the delivery, but still had a late delivery and the salesperson could not identify the exact delivery time to customers. In addition, the 3PL officers did not satisfy the delivery schedule since it had long delivery time. The other major problem was the high turnover rate of logistics department officer. Hence, the new officer needed to be trained. There was no systematic delivery process and all decision in solving the problem based on the skill and experience. For the same service zone, the 3PLs charges the transportation cost at a fixed cost per shipment. Hence, to reduce the transportation cost, it is to minimize the number of shipment or number of trips. There was a customer requirement with 100% service level. The delivery time was from 5 a.m. to 3 p.m. In addition, some customers could receive products from 5 a.m. to 8 a.m. only. Hence, there were at most two customer groups can be allocated in the same trip. This becomes the time windows constraint of the delivery. Then, Ongarj and Ongkunaruk (2013) formulated an integer programming (IP) to solve a bin packing problem (BPP) with time windows constraint. Their objective was to minimize the number of vehicles used by 3PLs. However, they left the routing problem to the drivers' responsibility.

The research related to Vehicle Routing Problem with Time Windows (VRPTW) were stated in Solomon (1983) and Solomon (1987), Desrochers et al. (1992) Bent and Van Hentenryck (2004), Azi et al. (2007). Kolen et al. (1987) used the branch and bound technique to solve a VRPTW with node number (i.e. retailer number) ranging from 6 to 15. For 6 nodes, the computer took one minute to find the solution, but for 12 nodes, the computer was unable to solve the problem. Later, Thangiah et al. (1994) developed a λ-interchange local search descent (LSD) method that used a systematic insertion and swapping of customers between routes, defined as λ-interchange operators. Due to computation burden, only 1-interchange and 2-interchange were commonly used. Hence, it allowed at most two customers to be inserted or swapped at one time. Then, Sam (1995) studied the VRPTW problem to reduce delivery costs with calculated only the travel time from one customer to another customer by separated waiting time and service time. In addition, he allowed to deliver product before or after time windows by adding a penalty in the objective function. Next, Dondo et al. (2003) presented a novel Mixed Integer Linear Programming (MILP) for the m-depot heterogeneous-fleet VRPTW problem which determined both the optimal vehicle route and the fleet size by choosing the best set of preceding nodes for each pick-up point. Then, Dondo and Cerda (2007) presented a three-phase heuristics for the multi-depot routing problem with time windows and heterogeneous vehicles. It has been derived from embedding a heuristic-based clustering algorithm within a VRPTW optimization framework. They proposed an MILP mathematical model for the VRPTW which could solve at most 25 nodes to optimality. To overcome this limitation, a preprocessing stage clustering nodes was initially performed to yield a more compact cluster-based MILP problem formulation. A hierarchical hybrid procedure involving one heuristic and two algorithmic phases was developed. Phase I identified a set of cost-effective feasible clusters while Phase II assigned clusters to vehicles and sequenced them on each routing by the cluster-based MILP formulation. Then, Phase III solved a small MILP within clusters and scheduled a vehicle arrival times at customer locations for each routing. Later, Cetinkaya et al. (2013) proposed a Mixed Integer Programming (MIP) formulation and a Memetic Algorithm (MA) for a variant of the vehicle routing problem, called Two-Stage Vehicle Routing Problem with Arc Time Windows (TS_VRP_ATW) problem. Experimental results indicated that the proposed MIP formulation gave the optimal solutions for small and medium-size test problems, and some large-size test problems. Moreover, the proposed MA achieved good quality solutions for the test problems in a short computation time. Then, they concluded that the problem with up to 50 nodes should be solved using the proposed MIP formulation. Otherwise, they recommended that it should be solved using proposed MA.

In addition, there were several researchers solved VRPTW using heuristics such as Holland (1975) developed the GA that coded the VRPTW solutions in forms of bit strings and initialized a population of random chromosome. Fitting chromosomes were then selected to undergo a crossover and mutation process, as to produce children which were different from the parents. This process was continued until a fixed number of generations has been reached or the evolution has converged. Next, Solomon (1983) developed a few heuristics for solving the VRPTW, including saving method, nearest neighbor, insertion, and sweeping, which the insertion method consistently gave very good results. Later, El-Sherbeny (2001) solved the VRPTW problem by a multi-objective simulated annealing (MOSA) method. Three categories of objectives were discussed: concerning the vehicle used (number of vehicles, number of covered/uncovered vehicles), concerning time (total duration of the routes, the homogeneity of the duration of the routes, working time not used, total waiting times due to time windows constraints), and concerning the flexible duration of the routes. Then, Beatrice et al. (2006) presented VRPTW problems with many objectives (Multi-Objective Problem) by finding minimum number of vehicles and the total cost of transportation or the shortest distance by genetic algorithm (GA) and pareto ranking, which can be used to solve the problem efficiently. Such methods can be applied to the VRPTW problem number of customers in more than 100 nodes.

In this paper, we formulated a mixed integer programming (MIP) to solve the VRPTW problem of the case study. Then, we compared our result with that of Ongarj and Ongkunaruk (2013).

2. METHODOLOGY

We used the data such as order quantity, number and location as well as the time requirement of customers, transportation cost and number of trucks from Ongarj and Ongkunaruk (2013). To reduce the size of problem, the company group the customers located in the same location in to the same group. Then, it divided the group customer locations into six zones as shown in Figure 1. Then, we proposed a mixed integer programming (MIP) of vehicle routing with time windows and solved the problem of six zones by an optimization package, IBM ILOG CPLEX version 12.4. Finally, we compared the results with that of current and Ongarj and Ongkunaruk (2013).

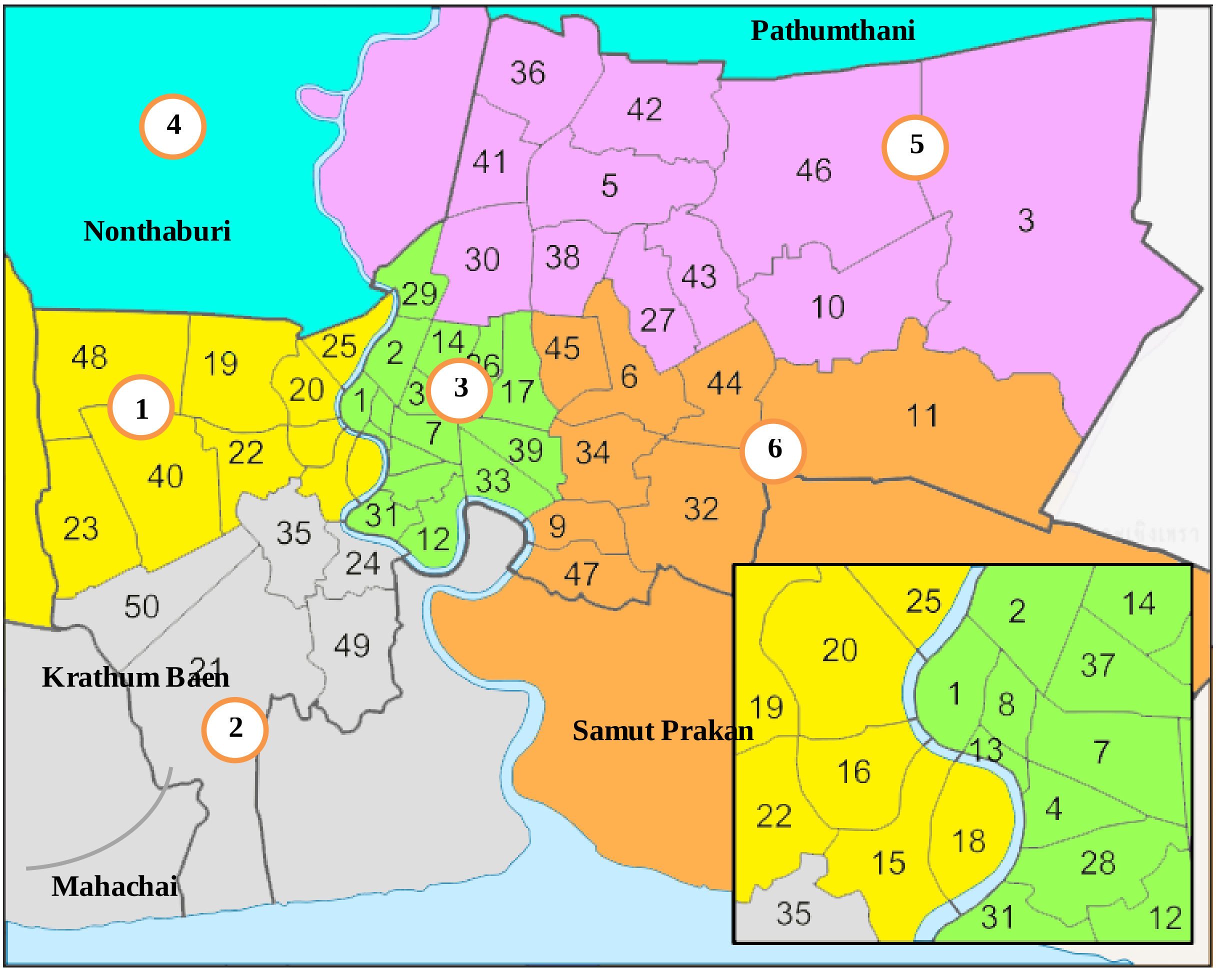

Figure 1: The zoning of the customers in Bangkok and metropolitan area of the company

3. MODEL FORMULATION

3.1 Subscripts and Set

i = customer index, I = a set of customers = 2…N+1
h, g = point index, H = a set of locations = 1…N+1
k = truck index, K = a set of vehicles = 1…M

3.2 Parameters

N = the number of customers
M = the number of vehicles
V_k = a variable cost of truck k
D_{gh} = a distance between point g and h
Q_i = the demand for product of customer i
C_k = a capacity of vehicle k
F_k = a fixed cost of vehicle k
P_k = a cost per mile of vehicle k

$$T_i = \begin{cases} 1 & \text{if customer } i \text{ has the time constraint before 08.00 p.m.} \\ 0 & \text{otherwise} \end{cases}$$

3.3 Decision Variables

$$x_{ghk} = \begin{cases} 1 & \text{if point g precedes h with vehicle k} \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ik} = \begin{cases} 1 & \text{if product for customer } i \text{ delivery by vehicle } k \\ 0 & \text{otherwise} \end{cases}$$

$$z_k = \begin{cases} 1 & \text{if vehicle } k \text{ is allocated} \\ 0 & \text{otherwise} \end{cases}$$

3.4 A Mathematical Model

$$Minimize \quad \sum_{k=1}^{M} F_k \cdot z_k + \sum_{k=1}^{M} V_k \sum_{i=2}^{N+1} Q_i y_{ik} + \sum_{k=1}^{K} \sum_{g=1}^{N+1} \sum_{h=1}^{N+1} P_k \cdot D_{gh} \cdot x_{ghk}$$

Subject to

$$\sum_{k=1}^{M}\sum_{h=1}^{N+1} x_{ihk} = 1 \qquad \forall i \in I, i \neq h \qquad (1)$$

$$\sum_{i=1}^{N+1} Q_i \sum_{h=1}^{N+1} x_{ihk} \leq C_k \qquad \forall k \in K \qquad (2)$$

$$\sum_{i=1}^{N+1} T_i y_{ik} \leq 2 \qquad \forall k \in K \qquad (3)$$

$$\sum_{g=1}^{N+1} x_{hgk} - \sum_{g=1}^{N+1} x_{ghk} = 0 \qquad \forall k \in K, h \in H \qquad (4)$$

$$a_g - a_h + N * x_{ghk} \leq N - 1 \qquad \forall g, h \in H, g \neq h, g \neq 1, k \in K \qquad (5)$$

$$\sum_{i=2}^{N+1} Q_i y_{ik} - C_k z_k \leq 0 \qquad \forall k \in K \qquad (6)$$

$$\sum_{h=1}^{N+1} x_{ihk} = y_{ik} \qquad \forall i \in H, i \neq h, k \in K \qquad (7)$$

$$x_{ghk} = binary \qquad \forall g, h \in H, k \in K \qquad (8)$$

$$y_{ik} = binary \qquad \forall i \in H, k \in K$$

$$z_k = binary \qquad \forall k \in K$$

$a_i =$ a real number associated with node i $\quad \forall i \in H$

The objective function was to minimize total cost of fixed cost of vehicles, variable cost of vehicles and total fuel cost used. Equation (1) implied that each group of customers would be allocated to one truck only. Equation (2) implied that the total products delivered by each truck were not over the truck capacity. Equation (3) implied that no more than two group of customers with time constraint were allocated in the same truck. Equation (4) implied that the vehicle should leave every point entered by the vehicle. Equation (5) was sub-tour elimination. Equation (6) linked the allocation of customer to the vehicle with the vehicle used. It implied that if there is an allocation of customer i to vehicle k, then vehicle k must be used. Equation (7) linked the allocation and routing components of the VRPTW. Finally, Equation (8) was specified that decision variables x, y, z is either 0 or 1 and a is non-negative real number. We solved the problem using parameter in Table 1 by CPLEX. The limited computational time was set to five minutes for each zone.

4. RESULTS AND DISCUSSIONS

After expert meeting in the company, the customer locations were categorized into six zones and each zone had different number of groups of customer as shown in Figure 1 and Table I. For example, in zone 1, there were 15 groups of customers with total 150 customers and demand 660 boxes per week. Then, there were four groups of customers who had a restriction in delivery time. Most zones had 100-150 customers with different demand and time window constraints. After solving the problem in each zone, we compared the results with that of current and Ongarj and Ongkunaruk (2013) methods as shown in Table 2 and 3. The weekly number of shipments was reduced from 26 to 20 trips accounting 9,413 baht per week or 489,476 baht per year. In addition, the third party logistics do not location routing, since solution is both bin packing and routing.

Table 1: The parameters of customers in 6 zones

Zone	# of group of customers in zone	# of Time Window customers	Total # of customers in each zone	Total demand for products (boxes/week)
1	15	4	150	660
2	11	3	116	655
3	17	4	131	463
4	17	3	103	1,055
5	13	2	80	825
6	16	4	110	430
Total	89	-	690	4,088

Table 2: The solution of proposed method

Zone	# of trips per week	Transportation Cost (baht/trip)	Transportation Cost (baht/week)
1	3	1,350	4,050
2	3	1,550	4,650
3	3	1,550	4,650
4	5	1,650	8,250
5	4	1,550	6,200
6	2	1,650	3,300
Total	20	-	31,100

Table 3: The comparison of solutions between the current and proposed methods

KPIs	Methods			% Improvement
	Company	Ongarj and Ongkunaruk (2013)	VRPTW	
# of trips/week	26	20	20	Reduced by 23%
Transportation Cost (baht/week)	40,513	31,100	31,100	Reduced by 23%
Planning Time (hours/day)	3	1	1	Reduced by 67%
OT (hours/day)	4	2	2	Reduced by 50%

5. CONCLUSIONS

This research solved the delivery problem of a case study company, which outsourced to the 3PL. However, it planned and scheduled the delivery for the 3PL. So far, the officer solved the problem based on the experienced and incurred the inefficient delivery system. Later, Ongarj and Ongkunaruk (2013) proposed an integer programming to solve the bin packing with time window which reduced the total cost. However, they left the routing problem to the driver responsibility. In this research, we proposed the mixed integer programming to solve the vehicle routing problem with time window. The result of the customer allocation was the same as they did, that is the total delivery was reduced by 23% or 9,413 baht per week. The overtime due to manual scheduling is reduced by 50% or two hours per day. However, we saved the time that the drivers have to figure out the routing problem. In the future, if the number of customers is increasing, then the problem becomes more difficult to solve optimally in a short time since VRPTW has been classified as an NP-hard problem (Winston et al., 2009). Hence, our future research can implement heuristics such as a genetic algorithm to solve the large problem in a short time.

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