MULTI-OBJECTIVES FINITE CAPACITY SCHEDULING OF MAKE-AND-PACK PRODUCTION WITH OPTIONS TO ADJUST PROCESSING TIME

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Abstract
Make-and-pack production is characterized by two stages of production namely, “make-stage” and “pack-stage” (Honkomp et al. 2000). Each stage consists of parallel processing units. In make-stage, raw materials are converted into final products by batch processing. Then, the final products are packed into containers in pack-stage. This paper develops finite capacity scheduling (FCS) system of make-and-pack production with multi-objectives and options to adjust processing time (OAPT). Multi-objectives including minimizations of total tardiness, total earliness, total flow time, and total processing costs are conflicting and a compromised solution is needed. Moreover, the processing time can be adjusted by adding some special chemicals. This paper proposes mixed-integer linear programming models to determine the compromised solution by using weighted average of satisfaction levels (WASL) of all objectives as performance measure. The proposed compromised solution method consists of three steps, (1) determining the best and worst values of each objective, (2) determining the initial compromised solution of all objectives when OAPT is not included in the model, and (3) determining the compromised solution with OAPT. The effect of chemical costs to the OAPT is evaluated. The results showed that chemical costs have significant effect to the OAPT. When the chemical cost is increased the OAPT is reduced and then the improvement of performance measure is also less. The proposed FCS system offered a compromised solution between conflicting objectives.

Keywords: make-and-pack production, mixed-integer linear programming, multi-objectives, compromised solution, adjustable processing times

1. INTRODUCTION

Scheduling system is the decision-making process that attempts to optimize one or more objectives by allocating limited resources to competing jobs over time. The developed scheduling systems typically decided on the assignment of products to processing units, and the sequencing and timing of jobs on each unit. Then the batch selection and batch sizing (batching) decisions are introduced in the scheduling system. In the chemical industry, make-and-pack production is classified as multistage processes where batches are processed in a sequence of stages from make-stage to pack-stage. Honkomp et al. (2000) presented the case study of make-and-pack production process which is characterized by two stages of production namely, “make-stage” and “pack-stage”. Each stage consists of parallel processing

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units. In make-stage, raw materials are converted into final products by batch processing. Then, the final products are packed into containers in pack-stage. The decisions include batch selection and sizing (batching), assignment, sequencing, and timing decisions. Up to now, several methods have been developed and proposed for scheduling these production processes. For some existing methods, the batching decisions are performed in advance of scheduling decisions. These methods often result in suboptimal solution. For other methods, batching and other scheduling decisions are performed simultaneously. Goals of scheduling systems focus on either single objective or a combination of objectives. When objectives are conflicting, a compromised solution is needed.

The purpose of this paper is to develop a finite capacity scheduling system of make-and-pack production that can provide the compromised solution for multi-objectives. The formulated MILP models are used to determine the performance measures of the system. The options to adjust processing time (OAPT) are then introduced to the MILP models to improve the system performances after the initial compromised solution is determined.

The paper is organized as follows. In section 2, the developments of scheduling system of multistage production processes that are related to make-and-pack production are reviewed. The development of model formulations is presented in section 3. Section 4 illustrates the method for determining the compromised solution of the scheduling system. A case study for evaluating the proposed scheduling system is given in section 5. In section 6, results are discussed. Finally, section 7 presents the conclusions of the paper and possible further studies.

2. LITERATURE REVIEWS

In this section, the developments of scheduling system that are related to make-and-pack production processes are reviewed. In the reviews, the developments of scheduling system with mixed-integer linear programing (MILP) models are the most concerned because this paper focuses on this approach. The scheduling systems are divided into two categories according to decisions-making during scheduling processes. First, the batching is independently decided from other scheduling decisions. Second, batching is simultaneously decided with scheduling decisions.

Fündeling and Trautmann (2006) proposed a priorities-based heuristic to compute schedule for make-and-pack production. The sequence of batches is determined by the technological constraints. An MILP model was proposed with block-planning concept by Günther et al. (2006). Méndez and Cerdá (2002) developed MILP formulation that is applied for scheduling of make-and-pack continuous production plant. In these approaches, batch size and sequence of batches are determined in advance. Baumann and Trautmann (2011) also developed scheduling system for a make-and-pack production with a continuous-time MILP model. Batch size is assumed to be known in advance. The assignment, sequencing, and timing of batches are determined simultaneously. Then the model was modified to cover all the technological constraints (Baumann and Trautmann 2013).

Méndez et al. (2006) has reviewed the optimization methods for short-term scheduling of batch processes and introduced the optimization approaches for the different problem types, focusing on both discrete and continuous-time models. Some approaches presented the separated decisions between batching and scheduling but others considered the batching and scheduling simultaneously. The simultaneous batching and scheduling decisions with MILP formation models were presented by Prasad et al. (2006), Prasad and Maravelias (2008), and Sundaramoorthy and Maravelias (2008a, 2008b). The methods that were previously developed involved single objective optimization. However, some of those methods mentioned alternative objectives where those are optimized separately. Prasad et al. (2006) suggested multiple factors in the objective function of the model but those factors are not conflicting with each other.

3. THE MODEL FORMULATIONS

In this section, there are descriptions of notations, problem statement and assumptions of make-and-pack production processes, parameters preparation, constraints, and objective functions of models.

3.1 Notations

Sets and Subsets

$\mathbf{F}$	Set of factors where $\mathbf{F} = \{TEN, TTN, TFT, TPC\}$
$\mathbf{I}$	Set of orders where $\mathbf{I} = \{1,2,..,\lvert\mathbf{I}\rvert\}$
$\mathbf{J}$	Set of units where $\mathbf{J} = \{1,2,..,\lvert\mathbf{J}\rvert\}$
$\mathbf{J}_s$	Set of units in stage s where $\mathbf{J} = \mathbf{J}_1 \cup \mathbf{J}_2 \cup .. \cup \mathbf{J}_{\lvert s\rvert}$
$\mathbf{S}$	Set of stages where $\mathbf{S} = \{1,2,..,\lvert\mathbf{S}\rvert\}$
$\mathbf{JA}_{is}$	Set of allowable units for order i in stage s where $\mathbf{JA}_{is} = \mathbf{J}_s \setminus \mathbf{JF}_{is}$
$\mathbf{JF}_{is}$	Set of forbidden units for order i in stage s
$\mathbf{FP}$	Set of forbidden path between units j and j'
$\mathbf{L}$	Set of batches where $\mathbf{L} = \{1,2,..,LMAX\}$
$\mathbf{L}_i$	Set of batches of order i where $\mathbf{L}_i = \{1,2,..,Lmax_i\}$
$\mathbf{IL}$	Set of pairs of batches (i,l) and (i',l') that can be sequenced where $\mathbf{IL} = \left\{i,i',l \in \mathbf{L}_i, l' \in \mathbf{L}_{i'} : (i \neq i') \cup \big((i = i') \cap (l \neq l')\big)\right\}$

Parameters

Q_i	The demand of order i (kg)
r_i/d_i	The release/due time of order i (hr)
$bmin_j/bmax_j$	The minimum/maximum operational capacity of unit j (kg)
$Bmin_i/Bmax_i$	The minimum/maximum feasible batch size of order i (kg)

$Lmin_i/Lmax_i$	The minimum/maximum potential number of batches of order i (unitless)
$LMAX$	The maximum number of batches for all orders (unitless)
Ft_{ij}/Pt_{ij}	The fixed/proportional processing time of order i in unit j (hr/kg)
Fc_{ij}/Pc_{ij}	The fixed/proportional processing cost of order i in unit j ($/hr)
Min_f/Max_f	The minimum/maximum value of factor f (depend on factor f)
$MaxWASL$	The initial maximum weighted average of satisfaction levels (unitless)
M	The big M value (unitless)
Cc_i	The chemical cost per unit of order i ($/kg)
Pr_i	The reduction rate of proportional processing time of order i (unitless)
w_f	The assigned weight of satisfaction level for factor f (unitless)

Independent Variables

Z_{il}	The selection of batch (i,l) (binary) $Z_{il} = 1$ if batch (i,l) is selected.
X_{ilj}	The assignment of batch (i,l) to unit j (binary) $X_{ilj} = 1$ if batch (i,l) is assigned to unit j.
$Y_{ili'l's}$	The pairwise sequencing between batches (i,l) and (i',l') in stage s (binary) $Y_{ili'l's} = 1$ if batch (i,l) is processed before (not necessary immediately before) batch (i',l') in unit j of stage s.
AC_{ilj}	The addition of chemical to batch (i,l) in unit j (binary) $AC_{ilj} = 1$ if chemical is added to batch (i,l) in unit j.
Ba_{il}	The size of batch (i,l) (kg)
Bb_{ilj}	The size of batch (i,l) processed in unit j (kg)
F_{ils}	The finished time of batch (i,l) in stage s (hr)

Dependent Variables

$Earl_{il}$	The earliness of batch (i,l) (hr)
$Tard_{il}$	The tardiness of batch (i,l) (hr)
$Flow_{il}$	The flow time of batch (i,l) (hr)

Performance Measures

TEN	The total earliness (hr)
TTN	The total tardiness (hr)
TFT	The total flow time (hr)
TPC	The total processing cost ($)
SL_f	The satisfaction level of factor f (unitless)
$WASL$	The weighted average of satisfaction levels (unitless)

3.2 Problem Statement and Assumptions

The model formulations are based on the following characteristics of make-and-pack production processes. (1) There is a set of orders with demand quantity, release and due times. (2) There is a set of processing units with minimum/maximum operational capacities, fixed/proportional processing times, and fixed/proportional processing costs. (3) There is a set of stages and in each stage there is a set of parallel (identical or non-identical) processing units. (4) There are sets of allowable units and forbidden units for each order in each stage, and forbidden path between processing units of each stage for all orders.

Assumptions are as follows. (a) Changeover time is not sequence dependent. Thus it is assumed to be part of the processing time. (b) Quality release time is not accounted; the successor operation can be started immediately if there is available processing unit. (c) All of the raw materials are available in sufficient quantity. (d) The storage tanks for products are unlimited. (e) All batches visit all stages. (f) All of the operations are non-preemptive.

3.3 Parameters Preparation

When minimum ($bmin_j$) and maximum ($bmax_j$) capacities of each processing unit, and demand (Q_i) of each order are given, the parameters of minimum ($Bmin_i$) and maximum ($Bmax_i$) feasible batch sizes, and minimum ($Lmin_i$) and maximum ($Lmax_i$) numbers of batches and maximum ($LMAX$) number of batch for all orders can be predetermined and used as parameters of models (eqs. 1–5).

$$Bmin_i = \max_{s \in \mathbf{S}} \left[\min_{j \in \mathbf{JA}_{is}} \left(bmin_j \right) \right];\ \forall i \in \mathbf{I} \tag{1}$$

$$Bmax_i = \min_{s \in \mathbf{S}} \left[\max_{j \in \mathbf{JA}_{is}} \left(bmax_j \right) \right];\ \forall i \in \mathbf{I} \tag{2}$$

$$Lmin_i = \lceil Q_i / Bmax_i \rceil;\ \forall i \in \mathbf{I} \tag{3}$$

$$Lmax_i = \lceil Q_i / Bmin_i \rceil;\ \forall i \in \mathbf{I} \tag{4}$$

$$LMAX = \max_{i \in \mathbf{I}} (Lmax_i) \tag{5}$$

3.4 Constraints

The MILP formulation models of make-and-pack production are constrained by conditions as follows.

Condition 1: Demand Satisfaction. The production quantity must satisfy the customer demand for all orders (eq. 6).

$$\sum_{l \in \mathbf{L}_i} Ba_{il} = Q_i;\ \forall i \in \mathbf{I} \tag{6}$$

Condition 2: Batch Selection and Batch Assignment. If batch (i, l) is selected, it must be assigned to only one processing unit j in each stage s (eq. 7). Then size of batch (i,l) must be between the minimum and maximum operational capacities of that processing unit (eqs. 8, 9).

$$Z_{il} = \sum_{j \in \mathbf{JA}_{is}} X_{ilj}\,;\ \forall i \in \mathbf{I}, l \in \mathbf{L}_i, s \in \mathbf{S} \tag{7}$$

$$Ba_{il} = \sum_{j \in \mathbf{JA}_{is}} Bb_{ilj}\,;\ \forall i \in \mathbf{I}, l \in \mathbf{L}_i, s \in \mathbf{S} \tag{8}$$

$$bmin_j X_{ilj} \le Bb_{ilj} \le bmax_j X_{ilj};\ \forall i \in \mathbf{I}, l \in \mathbf{L}_i, s \in \mathbf{S}, j \in \mathbf{JA}_{is} \tag{9}$$

Condition 3: Symmetry Breaking Purposes. This condition is used to restrict the selection and sizing of potential batches. For order i, a smaller batch number must be selected before a larger batch number can be selected (eq. 10). The batch size of a larger batch number is not allowed to exceed that of a smaller batch number (eq. 11).

$$Z_{i(l-1)} \ge Z_{il};\ \forall i \in \mathbf{I}, l \in \mathbf{L}_i \setminus \{1\} \tag{10}$$

$$Ba_{i(l-1)} \ge Ba_{il};\ \forall i \in \mathbf{I}, l \in \mathbf{L}_i \setminus \{1\} \tag{11}$$

Condition 4: Batch Sequence. When two batches (i,l) and (i',l') are processed in the same unit j in stage s, both batches have to follow the sequence either batch (i,l) or (i',l') is processed first because both batches cannot be processed in the same unit at the overlapped time (eq. 12).

$$X_{ilj} + X_{i'l'j} - 1 = Y_{ili'l's} + Y_{i'l'ils};\ \forall (i,l,i',l') \in \mathbf{IL}, i \le i', s \in \mathbf{S}, j \in \mathbf{JA}_{is} \cap \mathbf{JA}_{i's} \tag{12}$$

Condition 5: Non-Overlapping Processing Times of Batches. In the same stage s when the batch (i',l') is sequenced to process after the batch (i,l) the finished time of batch (i',l') is after batch (i,l) is finished plus the processing times of batch (i',l') in that stage (eq. 13). Between two consecutive stages, the finished time of batch (i,l) in stage $(s+1)$ is after it is finished from stage s plus its processing times in stage $(s+1)$ (eq. 14). The finished time of batch (i,l) in the first stage is after its release time plus its processing time in the first stage (eq. 15).

$$F_{i'l's} \ge F_{ils} + \sum_{j \in \mathbf{JA}_{i's}} \left(Ft_{i'j} X_{i'l'j} + Pt_{i'j} Bb_{i'l'j}\right) + M(1 - Y_{ili'l's});$$
$$\forall (i,l,i',l') \in \mathbf{IL}, s \in \mathbf{S} \tag{13}$$

$$F_{il(s+1)} \ge F_{ils} + \sum_{j \in \mathbf{JA}_{i(s+1)}} \left(Ft_{ij} X_{ilj} + Pt_{ij} Bb_{ilj}\right);\ \forall i \in \mathbf{I}, l \in \mathbf{L}_i, s < |\mathbf{S}| \tag{14}$$

$$F_{il1} \ge r_i Z_{il} + \sum_{j \in \mathbf{JA}_{i1}} \left(Ft_{ij} X_{ilj} + Pt_{ij} Bb_{ilj}\right);\ \forall i \in \mathbf{I}, l \in \mathbf{L}_i \tag{15}$$

Condition 6: Forbidden Units and Forbidden Path. The batch (i,l) is not allowed to be assigned to forbidden units for order i in stage s (eq. 16). The batch (i,l) is allowed to be assigned to at most one processing unit of forbidden paths between two consecutive stages (eq. 17).

$$X_{ilj} = 0;\ \forall i \in \mathbf{I}, l \in \mathbf{L}_i, s \in \mathbf{S}, j \in \mathbf{JF}_{is} \tag{16}$$

$$X_{ilj} + X_{ilj'} \leq Z_{il};\ \forall i \in \mathbf{I}, l \in \mathbf{L}_i, (j,j') \in \mathbf{FP} \tag{17}$$

Condition 7: Minimum Numbers of Batch Selection. At least the minimum numbers of batches must be selected to satisfy (eq. 14).

$$Z_{il} = 1;\ \forall i \in \mathbf{I}, l \leq Lmin_i \tag{18}$$

Condition 8: Earliness, Tardiness, and Flow Time Computations. When either earliness or tardiness is involved in the objective function, the earliness and tardiness can be computed in the model (eq. 19). However, when earliness and tardiness are both excluded, these values have to be computed outside the model (eq. 19a, 19b). Flow time of batch (i,l) is the different between finished time of last stage and started time of first stage (eq. 20).

$$Earl_{il} - Tard_{il} = d_i Z_{il} - F_{il|\mathbf{S}|};\ \forall i \in \mathbf{I}, l \in \mathbf{L}_i \tag{19}$$

$$Earl_{il} = \max\left(0, d_i Z_{il} - F_{il|\mathbf{S}|}\right);\ \forall i \in \mathbf{I}, l \in \mathbf{L}_i \tag{19a}$$

$$Tard_{il} = \max\left(0, F_{il|\mathbf{S}|} - d_i Z_{il}\right);\ \forall i \in \mathbf{I}, l \in \mathbf{L}_i \tag{19b}$$

$$Flow_{il} = F_{il|\mathbf{S}|} - \left[F_{il1} - \sum_{j \in \mathbf{JA}_{i1}} \left(Ft_{ij} X_{ilj} + Pt_{ij} Bb_{ilj}\right)\right];\ \forall i \in \mathbf{I}, l \in \mathbf{L}_i \tag{20}$$

Condition 9: Redundant Variable Eliminations. The redundant variables must be eliminated (eqs. 21–24).

$$Z_{il}, Ba_{il}, Earl_{il}, Tard_{il}, Flow_{il} = 0;\ \forall i \in \mathbf{I}, l \geq Lmax_i \tag{21}$$

$$X_{ilj}, Bb_{ilj} = 0;\ \forall i \in \mathbf{I}, l \geq Lmax_i, j \in \mathbf{J} \tag{22}$$

$$F_{ils} = 0;\ \forall i \in \mathbf{I}, l \geq Lmax_i, s \in \mathbf{S} \tag{23}$$

$$Y_{ili'l's} = 0;\ \forall (i,l,i',l') \notin \mathbf{IL}, s \in \mathbf{S} \tag{24}$$

Condition 10: Binaries and Non-Negativities. The binary and non-negativity conditions are also expressed (eqs. 25, 26).

$$Z_{il}, X_{ilj}, Y_{ili'l's} = \{0,1\} \tag{25}$$

$$Ba_{il}, Bb_{ilj}, F_{ils}, Earl_{il}, Tard_{il}, Flow_{il} \geq 0 \tag{26}$$

Condition 11: The OAPT. The OAPT is included in the model after the batches are selected and assigned to processing units. The processing times of batches are adjusted depending on the options to add special chemical into processing batches. The equations in condition 5 are modified with the term of proportional processing time reduction (eqs. 27–29). Flow time in condition 8 is also adjusted accordingly (eq. 30).

The chemical can be added to the batch (i, l) when it is processed in assigned units of make-stage. Therefore, in unassigned units and for all operations that are not in make-stage chemical cannot be allowed to be added to the batch (i, l) (eq. 31).

$$F_{i'l's} \geq F_{ils} + \sum_{j \in \mathbf{JA}_{i's}} \left[Ft_{i'j}X_{i'l'j} + Pt_{i'j}Bb_{i'l'j}\left(1 - Pr_{i'}AC_{i'l'j}\right)\right] + M(1 - Y_{ili'l's}); \quad \forall (i, l, i', l') \in \mathbf{IL}, s \in \mathbf{S} \tag{27}$$

$$F_{il(s+1)} \geq F_{ils} + \sum_{j \in \mathbf{JA}_{i(s+1)}} \left[Ft_{ij}X_{ilj} + Pt_{ij}Bb_{ilj}\left(1 - Pr_iAC_{ilj}\right)\right]; \ \forall i \in \mathbf{I}, l \in \mathbf{L}_i, s < |\mathbf{S}| \tag{28}$$

$$F_{il1} \geq r_iZ_{il} + \sum_{j \in \mathbf{JA}_{i1}} \left[Ft_{ij}X_{ilj} + Pt_{ij}Bb_{ilj}\left(1 - Pr_iAC_{ilj}\right)\right]; \ \forall i \in \mathbf{I}, l \in \mathbf{L}_i \tag{29}$$

$$Flow_{il} = F_{il|\mathbf{S}|} - \left[F_{il1} - \sum_{j \in \mathbf{JA}_{i1}} \left[Ft_{ij}X_{ilj} + Pt_{ij}Bb_{ilj}\left(1 - Pr_iAC_{ilj}\right)\right]\right]; \ \forall i \in \mathbf{I}, l \in \mathbf{L}_i \tag{30}$$

$$AC_{ilj} = 0; \quad \forall i \in \mathbf{I}, l \in \mathbf{L}_i, j \in \mathbf{j}: X_{ilj} = 0 \cup (j \notin \text{MakeStage}) \tag{31}$$

3.5 Objective Functions of Models

Individual Objectives

There are four factors that are minimized individually to measure individual performance including total earliness (TEN), total tardiness (TTN), total flow time (TFT), and total processing cost (TPC). These factors are expressed as follows (eqs. 32–35).

$$TEN = \sum_{i \in \mathbf{I}} \sum_{l \in \mathbf{L}_i} Earl_{il} \tag{32}$$

$$TTN = \sum_{i \in \mathbf{I}} \sum_{l \in \mathbf{L}_i} Tard_{il} \tag{33}$$

$$TFT = \sum_{i \in \mathbf{I}} \sum_{l \in \mathbf{L}_i} Flow_{il} \tag{34}$$

$$TPC = \sum_{i \in \mathbf{I}} \sum_{l \in \mathbf{L}_i} \sum_{s \in \mathbf{S}} \sum_{j \in \mathbf{JA}_{is}} \left(Fc_{ij}Ft_{ij}X_{ilj} + Pc_{ij}Pt_{ij}Bb_{ilj}\right) \tag{35}$$

Compromised Objective

Weighted average of satisfaction level ($WASL$) of all four factors is maximized to measure compromised performance (eq. 36). The satisfaction level (SL_f) of each factor is computed by eq. (37) for minimized factors and eq. (38) for maximized factors.

$$WASL = \sum_{f \in \mathbf{F}} w_f SL_f \tag{36}$$

$$SL_f = \frac{Max_f - f}{Max_f - Min_f}; \quad \forall f \in \mathbf{F} \tag{37}$$

$$SL_f = \frac{f - Min_f}{Max_f - Min_f}; \quad \forall f \in \mathbf{F} \tag{38}$$

When the OAPT is included in the model, the addition of chemical costs and reduction of proportional costs are accounted in the total processing cost expression (eq. 39). The chemical is added to a processing batch to improve the performance measure of the system. Therefore, performance measure after adding chemical must be greater or equal to its initial value (eq. 40).

$$TPC = \sum_{i \in \mathbf{I}} \sum_{l \in \mathbf{L}_i} \sum_{s \in \mathbf{S}} \sum_{j \in \mathbf{JA}_{is}} \left[Fc_{ij} Ft_{ij} X_{ilj} + Pc_{ij} Pt_{ij} Bb_{ilj} \left(1 - Pr_i AC_{ilj}\right) + Cc_i Bb_{ilj} AC_{ilj} \right] \tag{39}$$

$$WASL \geq MaxWASL \tag{40}$$

Note that eqs. (1)–(18), (21)–(26), and (32)–(35) are from Prasad and Maravelias (2008) and Sundaramoorthy and Maravelias (2008a, 2008b). Original ideas of this paper include eqs. (19), (19a), (19b), (20), (27)–(31) and (36)–(40).

4. THE PROPOSED COMPROMISED SOLUTION METHOD

The proposed compromised solution method consists of three steps, (1) determining the best and worst values of each objective, (2) determining the initial compromised solution of all objectives when OAPT is not included in the model, and (3) determining the compromised solution with OAPT.

4.1 Determining the Best and Worst Values

Preemptive goal programming (PGP) was introduced in disassembly-to-order (DTO) problem by Massoud and Gupta (2010) to decide the multi-criteria. The PGP model assigns the priorities to objectives and optimizations are performed in the sequence of priorities. In this paper, the PGP is applied to determine the best and worst values of each individual objective. There are two goals in the proposed method for PGP models. The first goal is the optimization of each individual objective. Then optimal value is fixed and the second goal is the minimization of total tardiness (TTN). But if TTN is the first goal then the second goal is the minimization of total earliness (TEN). For each objective, the best value is the optimal value, and the worst value is the worst of computed values provided by the PGP models. The PGP formulations for all factors are summarized in Table 1.

Table 1: The summary of PGP formulated models

Factor	1st Goal	2nd Goal
TEN	Obj.: $\min TEN$ St.: eqs. 6–26, and 32–35.	Obj.: $\min TTN$, given $Z_{il}, X_{ilj}, Bb_{ilj}, Earl_{il}$ St.: eqs. 12–15, 19–21, 23–26, and 32–35.
TTN	Obj.: $\min TTN$ St.: eqs. 6–26, and 32–35.	Obj.: $\min TEN$, given $Z_{il}, X_{ilj}, Bb_{ilj}, Tard_{il}$ St.: eqs. 12–15, 19–21, 23–26, and 32–35.
TFT	Obj.: $\min TFT$ St.: eqs. 6–18, 20–26, and 32–35.	Obj.: $\min TTN$, given $Z_{il}, X_{ilj}, Bb_{ilj}, Flow_{il}$ St.: eqs. 12–15, 19–21, 23–26, and 32–35.
TPC	Obj.: $\min TPC$ St.: eqs. 6–18, 20–26, and 32–35.	Obj.: $\min TTN$, given $Z_{il}, X_{ilj}, Bb_{ilj}$ St.: eqs. 12–15, 19–21, 23–26, and 32–35.

4.2 Determining the Initial Compromised Solution

After obtaining the best and worst values from PGP models, the satisfaction level of each objective is computed by eqs. (37) and (38). The weights are assigned to each satisfaction level based on relative importance of those objectives. The $WASL$ is maximized to solve the compromised solution for all objectives. The model formulation consists of a compromised objective function ($\max WASL$) and constraints (eqs. 6–26, and 32–35).

4.3 Determining the Compromised Solution with OAPT

The initial compromised solution model was formulated without OAPT. The batch selection, batch sizing, and batch assignment decisions that were resulted from initial compromised solution model are accepted. Then the OAPT is applied to the compromised solution model to decide on sequencing and timing of each processing batch. The modified model formulation consists of a compromised objective function ($\max WASL$) and constraints (eqs. 12, 19, 21, 23–34, and 36–40).

5. A CASE STUDY

In this section, we considered a case of production plant that has six processing units and ten ordered demands. Three units are used for mixing (make-stage) and other three units are used for packing (pack-stage). The operational capacities, processing times, and processing costs of each unit are given in Table 2. Each order is provided with demand, release/due time, and forbidden units in each stage as shown in Table 3. The production system has no forbidden paths between processing units in make-stage and pack-stage.

From information in Tables 2 and 3, the minimum and maximum feasible batch sizes, and the minimum and maximum numbers of batches for each order can be determined in parameters preparation step (eqs. 1–5), and used as parts of input parameters in the models (Table 4).

Table 2: Operational capacities, processing times, and processing costs

	Stage 1 (make)			Stage 2 (pack)		
Unit j	1	2	3	4	5	6
$bmin_j$ (kg)	30	25	30	20	30	25
$bmax_j$ (kg)	50	35	45	45	50	45
For $\forall i \in \mathbf{I}$						
Ft_{ij} (hr/setup)	2.25	3.00	2.00	2.50	3.00	2.00
Pt_{ij} (hr/kg)	0.20	0.30	0.25	0.15	0.15	0.15
Fc_{ij} (\$/hr)	25.00	10.00	20.00	15.00	10.00	20.00
Pc_{ij} (\$/hr)	13.50	7.00	10.00	10.00	10.00	10.00

Table 3: The demands, release/due times, and forbidden units in each stage

Order i	1	2	3	4	5	6	7	8	9	10
Q_i (kg)	75	90	120	65	90	125	65	80	95	120
r_i (hr)	0	0	0	0	0	0	0	0	0	0
d_i (hr)	48	56	80	48	80	96	56	72	96	102
$\mathbf{JF}_{i1}$	3	3	3	2	2	2	-	-	-	-
$\mathbf{JF}_{i2}$	6	6	6	5	5	5	4	4	4	4

Note: no forbidden path between make-stage and pack-stage.

Table 4: The feasible batch sizes and batch numbers

Order i	1	2	3	4	5	6	7	8	9	10
$Bmin_i$ (kg)	25	25	25	30	30	30	25	25	25	25
$Bmax_i$ (kg)	50	50	50	45	45	45	50	50	50	50
$Lmin_i$	2	2	3	2	2	3	2	2	2	3
$Lmax_i$	3	4	5	3	3	5	3	4	4	5

The effect of PGP models to the performance will be evaluated. Then we assumed that weights for satisfaction levels are 15% for TEN, 50% for TTN, 15% for TFT, and 20% for TPC. The initial performance of compromised solution method will be compared to the performance of PGP models. When OAPT is used, the effect of cost-reduction-ratio (CRR) to the performance will be evaluated. The CRR is the ratio between chemical cost per 1kg of batch and percent reduction of processing time. The unit of CRR is "*dollar per one kilogram of batch per one percent reduction of processing time*". We assumed that the percent reduction is constant at 20% and the chemical costs are computed from a set of CRR values (Table 5). The models in the case study are solved by using CPLEX 12.4 on desktop computers with a

3.30 GHz Core(TM) i5-2500 CPU and 9.00 GB RAM running on 64-bit operating system of windows 7.

Table 5: The CRR values and computed chemical costs

CRR	0.025	0.030	0.035	0.040
Pr_i (%)	20.00	20.00	20.00	20.00
Cc_i ($/kg)	0.50	0.60	0.70	0.80

6. RESULTS AND DISSCUSSIONS

Table 6 shows that both single factor optimizations and the PGP models provide the same best performance for each factor. But the PGP models result in narrower gap between best and worst performances.

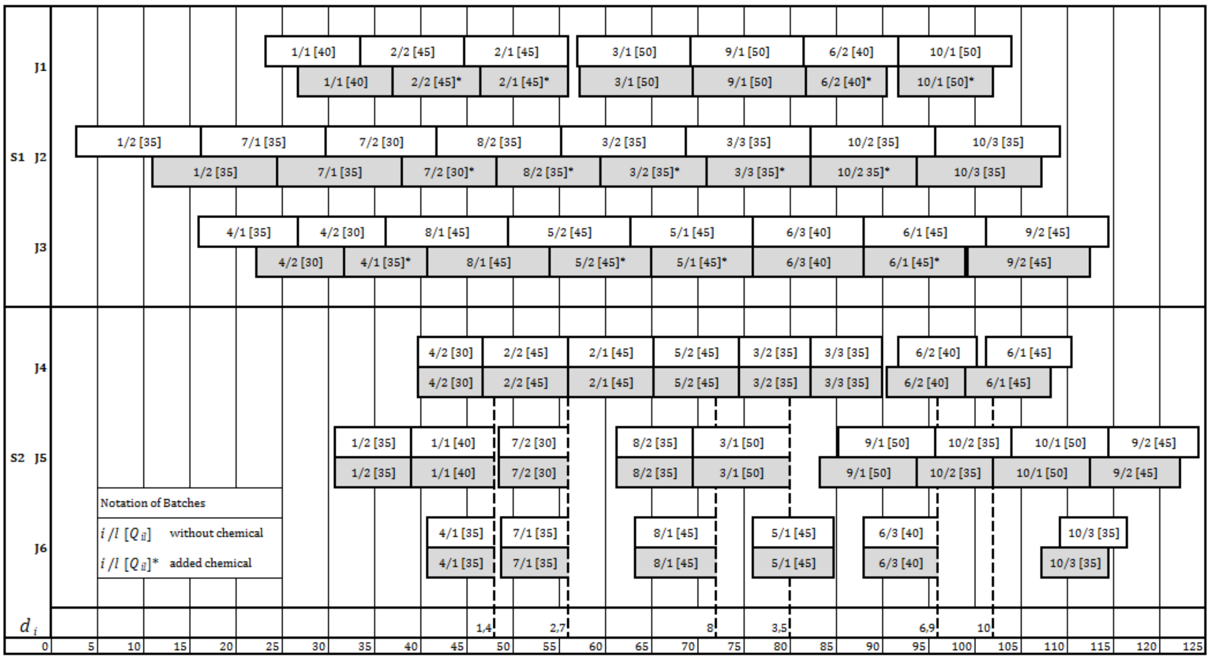
The compromised solution method does not provide the best performance for any factor but the WASL which is the system performance is the highest compared to all of the PGP models as shown in Table 7.

Table 6: Gap difference between single factor optimizations and the PGP models

Factor	Single factor optimizations			The PGP models			$\frac{Gap\ 1}{Gap\ 2}$
	Min_f	Max_f	$Gap\ 1$	Min_f	Max_f	$Gap\ 2$	
TEN (hr)	0.00	306.44	306.44	0.00	259.66	259.66	1.18
TTN (hr)	7.28	11300.50	11293.22	7.28	1227.75	1220.47	9.25
TFT (hr)	426.00	1901.62	1475.62	426.00	964.38	538.38	2.74
TPC ($)	5253.00	6071.25	818.25	5253.00	6071.25	818.25	1.00

Table 7: Performances for the PGP models and compromised solution (Comp.Sol.) method

Factor (w_f)	TEN then TTN	TTN then TEN	TFT then TTN	TPC then TTN	Comp.Sol. (Initial $WASL$)
TEN (hr) (0.15)	**0.00** (**1.000**)	259.66 (0.000)	73.50 (0.717)	166.75 (0.358)	18.50 (0.929)
TTN (hr) (0.50)	465.00 (0.625)	**7.28** (**1.000**)	1227.75 (0.000)	440.25 (0.645)	102.25 (0.922)
TFT (hr) (0.15)	964.38 (0.000)	656.03 (0.573)	**426.00** (**1.000**)	637.37 (0.607)	579.75 (0.714)
TPC ($) (0.20)	5761.50 (0.379)	5792.6 (0.341)	6071.25 (0.000)	**5253.00** (**1.000**)	5417.75 (0.799)
WASL	(0.538)	(0.654)	(0.258)	(0.667)	(**0.867**)



Note: shaded area means with OAPT at CRR = 0.030

Figure 1: Production scheduling gantt chart for Comp.Sol. method without and with OAPT

Figure 1 shows that the utilization of processing units in make-stage is very high for the compromised solution method. When the chemical is added to a processing batch in make-stage the processing time of that batch is reduced and the operation times are shifted. Sometimes, the sequence of batches in processing units is changed; for example, batches $(4, 1)$ and $(4, 2)$ in the third processing unit are switched.

Table 8: Performance measures of compromised solution: initial and with OAPT

Factor (w_f)	Comp.Sol. (Initial $WASL$)	Comp.Sol. with OAPT			
		CRR = 0.025	CRR = 0.030	CRR = 0.035	CRR = 0.040
TEN (hr) (0.15)	18.50 (0.929)	20.50 (0.921)	20.50 (0.921)	18.85 (0.927)	18.50 (0.929)
TTN (hr) (0.50)	102.25 (0.922)	88.20 (0.934)	90.75 (0.932)	103.30 (0.921)	102.25 (0.922)
TFT (hr) (0.15)	579.75 (0.714)	480.70 (0.898)	500.40 (0.862)	534.20 (0.799)	567.15 (0.738)
TPC ($) (0.20)	5417.75 (0.799)	5424.15 (0.791)	5476.15 (0.727)	5456.15 (0.752)	5431.05 (0.782)
PC ($) CC ($)	5417.75	4979.15 445.00	5164.15 312.00	5351.15 105.00	5403.05 28.00
WASL	(0.867)	(0.898)	(0.879)	(0.870)	(0.868)

Table 8 shows that when the CRR is increased, the actual processing cost (PC) is also increased but the chemical cost (CC) is reduced because less chemical is added to processing batches. The system performance (WASL) is also reduced. Therefore, when the chemical cost per unit is higher, the usage of chemical is less and then the system performance is lower to the initial performance.

7. CONCLUSIONS

We have presented the FCS system of make-and-pack production with multi-objectives and OAPT to determine the compromised solution for the system. From case study, the PGP models can narrow down the gap between best and worst performances of each objective when the best performances are the same to single factor optimizations. The compromised solution method results the highest WASL compared to the PGP models. This method also generates a production schedule with relatively high utilization of each processing unit in make-stage. When the OAPT is used, the performances are improved according to the chemical cost per unit. When the chemical cost per unit is higher the performance of the system is reduced toward its initial performance.

The limitation of the proposed method is that the production schedule is applied only for continuous-time production. There is only single level of chemical option to be added. For further the system should be modified to apply for discontinuous-time production that is used by most of small and medium production plants. The multi-level of chemical options to be added in the processing batch should be studied.

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