MULTI-SOURCING MULTI-PRODUCT SUPPLIER SELECTION: AN INTEGRATED FUZZY MULTI-OBJECTIVE LINEAR MODEL

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Abstract

Supplier selection is an important strategic supply chain design decision. It is always exposed to major risks and a number of uncertainties in the decision such as risks of not having sufficient raw materials to meet their fluctuating demand. These risks and uncertainty may be caused by natural disasters to man-made actions. Incorporating the uncertainty of demand and supply capacity into the optimization model results in a robust selection of suppliers. The fuzzy set theories can be employed due the presence of vagueness and imprecision of information. In addition, supplier selection is a Multi-Criteria Decision Making problem (MCDM) in which criteria has different relative importance. In order to select the best suppliers it is necessary to make a trade-off between these tangible and intangible factors some of which may conflict. This study focuses on a fuzzy multi-objective linear model to deal with the problem. The model is capable of incorporating multiple products with multiple suppliers (sourcing). The proposed model can help the Decision Makers (DMs) to find out the appropriate order to each supplier, and allows the purchasing manager(s) to manage the supply chain performance on cost, quality and service. The model is explained by an illustrative example, showing that the proposed approach can handle realistic situation when there is information vagueness related to inputs.

Keywords: Supplier selection, Fuzzy MCDM, Multi-sourcing, Multi-product

1. INTRODUCTION

1.1. Supplier Selection

Supplier selection and evaluation have been one of the major topics in production and operations management literature, especially in advanced manufacturing technologies and environment (Montwani, et al., 1999). The main objective of supplier selection processes is to reduce purchase risk, maximize overall value to the purchaser, and develop closeness and long-term relationships between buyers and suppliers, which is effective in helping the company to achieve Just-In-Time (JIT) production (Li et al., 1997). Additionally with the increase in use of Total Quality Management (TQM), the supplier selection question has become extremely important (Petroni, 2000).

Choosing the right method for supplier selection effectively leads to a reduction in purchase risk and increases the number of JIT suppliers and TQM production.

Supplier selection is a Multiple Criteria Decision Making (MCDM) problem, which is affected by several conflicting factors. Consequently, a purchasing manager must analyze the trade-off between the several criteria. MCDM techniques support the Decision Makers (DMs) in evaluating a set of alternatives (Amid et al., 2006). Supplier selection problem has become one of the most important issues for establishing an effective supply chain system. The purchasing manager must know a suitable method and use the best method from the different types of methods to select the right supplier. The supplier selection problem in a supply chain system is a group decision according to multiple criteria from which a number of criteria have been considered for supplier selection in previous and present decision models (Chen-Tung et al., 2006).

1.2. Uncertainty of Decision Making in Manufacturing

The main disadvantage of deterministic models is their incapability of handling randomness embedded in the real system. Decision making in real manufacturing requires considering multitude of uncertainty. Variations in human operator performance, inaccuracies of process equipment and volatility of environment condition are but just a few of these types of uncertainties. Internally, uncertainties may be caused by human, machine or systems related issues. External factors related to changes in demand or other exogenous factors (policy, market forces, competitive behaviors) can also inject uncertainty into the decisions.

Fuzzy logic (Zadeh, 1965, 1996, 1997) is an analysis method purposefully developed to incorporate uncertainty into a decision model. Fuzzy logic allows for including imperfect information no matter the cause. In essence fuzzy logic allows for considering reasoning that is approximate rather than precise. There are key benefits to applying fuzzy tools. Fuzzy tools provide a simplified platform where the development and analysis of models require reduced development time than other approaches. As a result, fuzzy tools are easy to implement and modify. Nevertheless, despite their user-friendly outlet, fuzzy tools have shown to perform just as or better than other soft approaches to decision making under uncertainties. These characteristics have made fuzzy logic and tools associated with its use to become quite popular in tackling manufacturing related challenges (Lee, 1996).

1.3. Single *vs* Multiple Sourcing Supplier Selection under Fuzzy Environment

Some of the above mentioned papers deal with single sourcing supplier selection in which one supplier can satisfy all buyers' need while more recent ones discussed multiple sourcing. With multiple sourcing, a buyer may purchase the same product(s) from more than one supplier. If the volume is large enough, demand requirements are split among several suppliers. Having additional suppliers may alleviate the situation when the supplier's production capacity is insufficient to meet a peak demand. Multiple sourcing also motivates suppliers to be price and quality competitive. Most purchasing professionals agree that when buyers use more than one supplier for a product, the buying firm generally will be protected in times of shortage (Zenz, 1987). For organizations that experience uneven demand, bottlenecks may occur if the supplier's production capacity is insufficient to meet a peak demand. Having additional suppliers alleviates this problem.

Ghodsypour and O'Brien (2001) have stated that only a few mathematical programming models have been published to this date those analyze supplier selection problems involving multiple sourcing with multiple criteria and with supplier's capacity constraints. Kumer et al. (2004) proposed fuzzy goal programming for the supplier selection problem with multiple sourcing that included three primary goals: minimizing the net cost, minimizing the net rejections and minimizing the net late deliveries, subject to realistic constraints regarding buyers' demand and vendors' capacity. In their proposed model, a weightless technique is used in which there is no difference between objective functions. In other words, the objectives are assumed equally important in this approach and there is no possibility for the DM to emphasize objectives with heavy weights. In real situation for supplier selection problem, the weights of criteria could be different and depend on purchasing strategies in a supply chain (Wang et al., 2004). For instance, Amid et al. (2006, 2009) developed a weighted additive fuzzy model for supplier selection problems to deal with imprecise inputs and the basic problem of determining weights of quantitative/qualitative criteria under conditions of multiple sourcing and capacity constraints. In the weighted additive model, there is no guarantee that the achievement levels of fuzzy goals are consistent with desirable relative weights or the DM's expectation (Chen and Tasi, 2001 and Amid et al., 2006). In their later paper, a weighted max-min fuzzy multi-objective model has been developed for the supplier selection problem to overcome the above problem. This fuzzy model enables the purchasing managers not only to consider the imprecise of information by also to take the limitations of buyer and supplier into account in calculating the order quantities from each supplier as well as matches the relative importance the objective functions (Amid et al., 2011).

1.4. Single *vs* Multiple Materials/Products Model

In product configuration, the finished product is usually composed of many parts. Each of those parts can be provided by various suppliers from different geographical locations. In order to enhance the product functions, the challenge of the configuration change is to find suitable part suppliers that provide quality components, and can effectively fulfill these requirements the best. In other words, based upon consumer or engineering requirements, an appropriate part supplier combination is required for a specific product in order to decide which supplier will provide which component. The question is what combination of part suppliers will best fulfill the requirements of both, low cost and high quality? It is the purpose of the 'supplier combination' to assess all of the potential part suppliers and determine the most superior combination.

Even with multiple sourcing, all above mentioned papers usually deal with a single material (product). However, only a few papers to our knowledge have been extended to cover multiple materials under some uncertainties. In this instance, the firm could work with a number of suppliers for its raw materials. Some of the raw materials have been supplied from multiple sources while some of the others have been supplied from single source. There have also been alternative suppliers for each raw material. Cebi and Bayraktar (2003) addressed the supplier selection problem with multiple sourcing and multiple raw materials. In their case study, within the conflicting objectives of the firm (Turkish food manufacturing firm) that are quality maximization, late order percentage minimization, purchasing cost minimization and also utilization maximization, 9 suppliers from 13 suppliers have been proposed to get the orders and the results have been found to be consistent and reliable by the management.

2. BASIC DEFINITION AND CALCULATION MODEL OF FACTORS

A positive trapezoidal number ñ can be defined as (n_1, n_2, n_3, n_4) shown in Figure 1 and the membership function $\mu_{\tilde{n}}(x)$ is expressed as: (Kaufmann and Gupta, 1991)

$$\mu_{\tilde{n}}(x) = \begin{cases} 0, & x < n_1, \\ \frac{x-n_1}{n_2-n_1}, & n_1 \le x \le n_2, \\ 1, & n_2 \le x \le n_3, \\ \frac{x-n_4}{n_3-n_4}, & n_3 \le x \le n_4 \\ 0, & x > n_4 \end{cases} \quad (1)$$

For a trapezoidal number if $n_1 = n_2$ then the number is called as triangular fuzzy number.

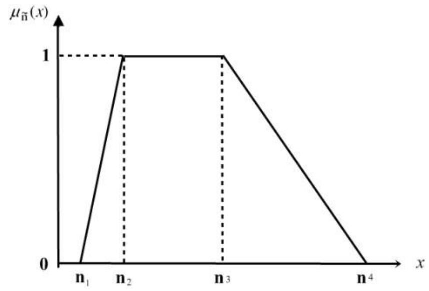

Figure 1: Trapezoidal number ñ

A linguistic variable is a variable whose values are expressed in linguistic terms. For example, if “temperature” is interested as a linguistic variable, then its term set could be “very low”, “low”, “comfortable”, “high” and “very high” (Zimmermann, 1993). In this paper, DMs use the linguistic values shown in Figure 2 to assess the weights of the factors in fuzzy multi-objective linear model.

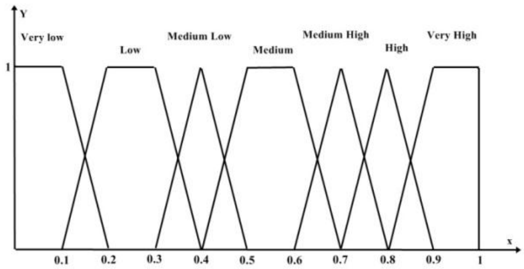

Figure 2: Linguistic variables for importance weight of each factor.

Let $\tilde{m}$ = (m_1, m_2, m_3, m_4) and ñ = (n_1, n_2, n_3, n_4) be two trapezoidal fuzzy numbers. Then the distance between them can be calculated by using the vertex methods as: (Chen, 2000)

$$d_v(\tilde{m},\tilde{n}) = \sqrt{\frac{1}{4}[(m_1 - n_1)^2 + (m_2 - n_2)^2 + (m_3 - n_3)^2 + (m_4 - n_4)^2} \tag{2}$$

Assume that a decision group has *B* decision makers as *b = 1, 2, ..., B* and considers a set of *j* criteria as *j= 1, 2, ..., n* for a supplier selection problem. Then, the aggregated fuzzy weights (w_j) of each criterion can be calculated as: (Chen et al., 2006)

$(\widetilde{w_j}) = (w_{j1}, w_{j2}, w_{j3}, w_{j4})$,

where

$$w_{j1} = \min{}_k \{w_{jb1}\},\ w_{j2} = \frac{1}{b}\Sigma_{b=1}^{B} w_{fb2},\quad w_{j3} = \frac{1}{j}\Sigma_{b=1}^{B} w_{jb3},\quad w_{j4} = \max{}_j \{w_{jb4}\}. \tag{3}$$

Similar to AHP and TOPSIS approaches and considering the linguistic variables (*lv*), Fuzzy Positive Ideal Rating (FPIR – A*) and fuzzy negative-ideal rating (FNIR – A^-) of a selection criterion can be defined as:

$A^* = lv^*$,

$A^- = lv^-$ (4)

According to the linguistic variables shown in Figure 2, FPIR and FNIR of a selection criteria can be expressed as respectively, "very high" (0.8, 0.9, 1.0, 1.0) and "very low" (0.0, 0.0, 0.1, 0.2). The distance between aggregated fuzzy weights (w_j) of each criterion and ideal ratings can be calculated by applying vertex method (2).

A closeness coefficient is determined to calculate the weights of each factor for the developed fuzzy multi-objective linear model.

$$CC_f = \frac{d_f^-}{d_f^* + d_f^-},\ f = 1, 2, \ldots, F, \tag{5}$$

where d_f^- is distance to FNIR, $d_f^*$is distance to FPIR.

By applying normalization to closeness coefficients obtained from (5), final weights (w_j) of each factor can be calculated as:

$$w_f = \frac{CC_f}{\Sigma_{f=1}^{F} CC_f}. \tag{6}$$

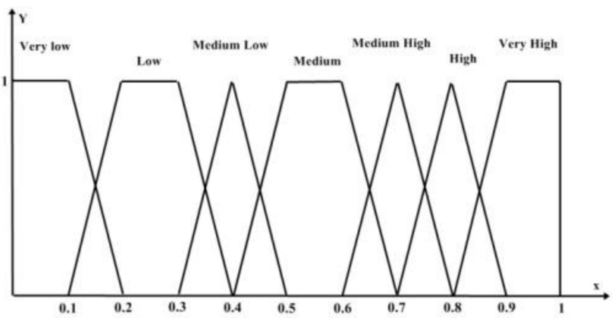

Figure 2: Linguistic variables for importance weight of each factor.

2.1 The Fuzzy Multi-Objective Supplier Selection Model for a Single Product

A general multi-objective model for the supplier selection problem for a single product can be stated as follows:

$$\min Z_1, Z_2, \ldots\ldots, Z_k \tag{7}$$

$$\max Z_{k+1}, Z_{k+2}, \ldots\ldots., Z_{k+n} \tag{8}$$

s.t.:

$$x \in X_d, X_d = \{x/g(x) \le b_r, r = 1,2,\ldots\ldots, m\} \tag{9}$$

where Z_1, Z_2, ..., Z_k are the negative objectives or criteria-like cost, late delivery, etc. and Z_{k+1}, Z_{k+2}, ..., Z_p are the positive objectives or criteria such as quality, on time delivery, after sale service and so on. X_d is the set of feasible solutions which satisfy the constraint such as buyer demand, supplier capacity, etc.

A typical linear model for supplier selection problems is min Z_1; max Z_2, Z_3 with

$$Z_1 = \sum_{i=1}^{n} P_i x_{i,} \qquad \text{(such as cost)} \tag{10}$$

$$Z_2 = \sum_{i=1}^{n} F_i x_i, \qquad \text{(such as quality)} \tag{11}$$

$$Z_3 = \sum_{i=1}^{n} S_i x_i \qquad \text{(such as on time delivery)} \tag{12}$$

s.t.:

$$\sum_{i=1}^{n} x_i \ge D, \tag{13}$$

$$x_i \le C_i, i = 1,2,\ldots,n \tag{14}$$

$$P_i x_{i,} \le U_i, i = 1,2,\ldots,n \tag{15}$$

$$x_i > 0, i = 1,2,\ldots,n \tag{16}$$

where D is demand over period, x_i is the number of units purchased from the i^{th} – supplier, P_i is per unit net purchase cost from supplier i, C_i is capacity of i^{th} supplier, U_i is the purchased budget from i^{th} supplier, F_i is percentage of quality level of i^{th} supplier, S_i is percentage of on time delivery of i^{th} supplier, n is number of suppliers.

Three objective functions – net price (10), quality (11) and delivery (12) – are formulated to minimize total monetary cost, maximize total quality and on time delivery of purchased items, respectively. Constraint (13) ensures that demand is satisfied. Constraint (14) means that order quantity of each supplier should be equal or less than its capacity. Constraint (15) represents the limitation of the purchased budget given from each supplier and constraint (16) prohibits negative orders.

2.2 The Fuzzy Supplier Selection Model

In this section, first the general multi-objective model for supplier selection is presented and then appropriate operators for this decision-making problem are discussed.

A general linear multi-objective model can be presented as:

Find a vector x written in the transformed form $x^T = [x_1, x_2, \ldots, x_n]$ which minimizes objective function Z_k and maximizes objective function Z_l with

$$Z_k = \sum_{i=1}^{n} c_{ki} x_i, k = 1, 2, \ldots, p, \tag{17}$$

$$Z_l = \sum_{i=1}^{n} c_{li} x_i, \ \ l = p+1, p+2, \ldots, q \tag{18}$$

and constraints:

$$x \in x_d, x_d = \{x / g(x) = \sum_{i=1}^{n} a_{ri} x_i \leq b_r, \ r = 1, 2, \ldots, m, x \geq 0\}, \tag{19}$$

where c_{ki}, c_{li},a_{ri} and b_r are crisp or fuzzy values.

Zimmermann (1987) has solved problem (17-19) by using fuzzy linear programing. He formulated the fuzzy linear program by separating every objective function Z_j into its maximum $Z_k^+$and minimum Z_l^- value by solving:

$$Z_k^+ = \max Z_k, x \in X_a, Z_k^- = \min Z_k, x \in X_d, \tag{20}$$

$$Z_l^+ = \max Z_l, x \in X_d, Z_l^- = \min Z_l, x \in X_a, \tag{21}$$

$Z_k^-, Z_l^+$are obtained through solving the multi-objective problem as a single objective using, each time, only one objective and $x \in X_d$ means that solutions must satisfy constraints while X_a is the set of all optimal solutions through solving as single objective.

Since for every objective function Z_j, its value changes linearly for $Z_j^-$to Z_j^+, it may be considered as a fuzzy number with the linear membership function $\mu_{zj}(x)$ as shown in Figure 3.

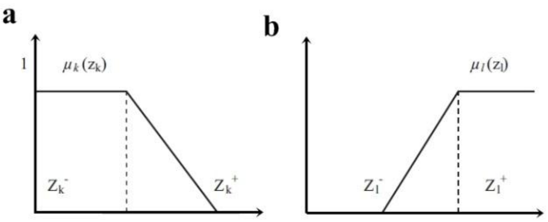

Figure 3: Objective function as fuzzy number: (a) min Z_k and (b) max Z_l.

It was shown that a linear programing problem (16-18) with fuzzy goal and fuzzy constraints may be presented as follows:

Find a vector x to satisfy:

$$\widetilde{Z_k} = \sum_{i=1}^{n} c_{ki} x_i \leq \sim Z_k^0, k = 1, 2, \ldots, p, \tag{22}$$

$$\widetilde{Z}_l = \sum_{i=1}^{n} c_{li} x_i \geq \sim Z_l^0,\ l = p+1, p+2, \dots, q \tag{23}$$

s.t.:

$$\widetilde{g}_l(x) = \sum_{i=1}^{n} a_{ri} x_i \leq \sim b_r,\ r = 1, 2, \dots, h \quad \textit{(for fuzzy constraints)}, \tag{24}$$

$$g_p(x) = \sum_{i=1}^{n} a_{pi} x_i \leq b_p, p = h+1, \dots, m \ \textit{(for deterministic constraints)}, \tag{25}$$

$$x_i \geq 0, i = 1, 2, \dots, n. \tag{26}$$

In this model, the sing $\sim$ indicates the fuzzy environment. The symbol $\leq \sim$ in the constraints set denotes the fuzzified version of $\leq$ and has linguistic interpretation "essentially smaller than or equal to" and the symbol $\geq \sim$ has linguistic interpretation "essentially greater than or equal to". Z_k^0 and Z_l^0 are the aspiration levels that the decision-maker wants to reach.

Assuming that membership functions, based on preference or satisfaction are linear, the linear membership for minimization goals (Z_k) and maximization goals (Z_l) are given as follows:

$$\mu_{zk}(x) = \begin{cases} 1 & for\ Z_k \leq Z_k^- \\ (Z_k^+ - Z_k(x))/(Z_k^+ - Z_k^-) & for\ Z_k^-(x) \leq Z_k(x) \leq Z_k^+,\ k = 1,2,\dots,p, \\ 0 & for\ Z_k \geq Z_k^+ \end{cases} \tag{27}$$

$$\mu_{zl}(x) = f(x) = \begin{cases} 1 & for\ Z_l \geq Z_l^+ \\ (Z_l(x) - Z_l^-)/(Z_l^+ - Z_l^-) & for\ Z_l^- \leq Z_l(x) \leq Z_l^+, l = p+1, p+2, \dots, q, \\ 0 & for\ Z_l \leq Z_l^- \end{cases} \tag{28}$$

The linear membership function for the fuzzy constraints is given as

$$\mu_{gr}(x) = \begin{cases} 1 & for\ g_r(x) \leq b_r, \\ 1 - \frac{g_r(x) - b_r}{d_r} & for\ b_r \leq g_r(x) \leq b_r + d_r, r = 1,2,\dots,h, \\ 0 & for\ g_r(x) \geq b_r + d_r, \end{cases} \tag{29}$$

d_r is the subjectively chosen constants expressing the limit of the admissible violation of the r^{th} inequalities constraints (tolerance interval). In the next section, some important fuzzy decision-making operators will be presented.

2.3 Decision Making Operators

First, the weighted additive method operator is discussed, which was used by Zimmermann (1987, 1993) for fuzzy multi-objective problems to assign different weights to various criteria.

In fuzzy programing modeling, using Zimmermann's approach, a fuzzy solution is given by the intersection of all the fuzzy sets representing either fuzzy objective or fuzzy constraints. The solution for all fuzzy objectives and h fuzzy constraints may be given as:

$$\mu_D(x) = \left\{\left\{\cap_{j=1}^{q} \mu_{zj}(x)\right\} \cap \left\{\cap_{r=1}^{h} \mu_{g_r}(x)\right\}\right\}. \tag{30}$$

The optimal solution (x*) is given by

$$\mu_D(x^*) = \max_{x\in X_d} \mu_D(x) = \max_{x\in X_d} \min \ [\max_{j=1,\dots,q} \mu_{Zj}(x), \min_{r=1,\dots,h} \mu_{g_r}(x)]. \qquad (31)$$

The convex fuzzy model proposed by Bellman and Zadeh (1970), Sakawa (1993) and the weighted additive model by Tiwari et al. (1987) is:

$$\mu_D(x) = \sum_{j=1}^{q} w_j \mu_{zj}(x) + \sum_{r=1}^{h} \beta_r \mu_{g_r}(x), \qquad (32)$$

$$\sum_{j=1}^{q} w_j + \sum_{r=1}^{h} \beta_r = 1, \beta_r \geq 0, \qquad (33)$$

where w_j and β_i are the weighting coefficients that present the relative importance among the fuzzy goals and fuzzy constraints. The following crisp single objective programing is equivalent to the above fuzzy model:

$$\max \sum_{j=1}^{q} w_j \, \mu_{zj}(x) + \sum_{r=1}^{h} \beta_r \mu_{g_r}(x), \qquad (34)$$

$$\sum_{j=1}^{q} w_j + \sum_{r=1}^{h} \beta_r = 1, w_j, \beta_r \geq 0, \qquad (35)$$

where w_j and β_i are the weighting coefficients that present the relative importance among the fuzzy goals and fuzzy constraints. The following crisp single objective programing is equivalent to the above fuzzy model:

$$\max \sum_{j=1}^{q} w_j \lambda_j + \sum_{r=1}^{h} \beta_r \gamma_r \qquad (36)$$

s.t.:

$$\lambda_j \leq \mu_{zj}(x), j = 1, 2, \dots, q, \qquad (37)$$

$$\gamma_r \leq \mu_{g_r}(x), r = 1, 2, \dots, h, \qquad (38)$$

$$g_p(x) \leq b_p, p = h + 1, \dots, m, \qquad (39)$$

$$\lambda_j, \gamma_r \in [0,1], j = 1, 2, \dots, q \ \ and \ r = 1, 2, \dots, h, \qquad (40)$$

$$\sum_{j=1}^{q} w_j + \sum_{r=1}^{h} \beta_r = 1, \ w_j, \beta_r \geq 0, \qquad (41)$$

$$x_i \geq 0, i = 1, 2, \dots, n. \qquad (42)$$

3. NUMERICAL EXAMPLE

The model algorithm with multiply products is illustrated through a numerical example.

The variables are:

Y_{ei} = "1" if supplier is chosien for raw material e, "0" otherwise
X_{ei} = amount of raw material e to be purchased from supplier i
λ_j = satisfaction level of criteria j

The constraints are:

Q_{eimin} = minimum order quantity from supplier *i* for raw material *e*
Q_{eimax} = maximum order quantity from supplier i for raw material *e*
S_{ei} = rate of perfect delivery of raw material e from supplier *i*
A_{ei} = rate of perfect quality raw of material e from supplier *i*
C_{ei} = unit purchasing of raw material *e* from supplier *i*
U_i = purchased budget from i^{th} supplier
max_j = maximum possible value of criteria *j*
min_j = minimum possible value of criteria *j*
n_e = number of supplier to be selected for raw material *e*

Objective function: $\text{Max}\sum_f^F W_e\lambda_{fe} + \sum_{r=1}^{h}\beta_r\gamma_r$
Subject to

$$\lambda_j \leq \frac{max_j - Z_j}{max_k - min_j} \quad ; \quad \forall j \in \min Z_j$$

$$\lambda_j \leq \frac{Z_j - max_j}{max_j - min_j} \quad ; \quad \forall j \in \max Z_j$$

Objective 1: $Z_{max} = \sum_e \sum_i S_{ei} * X_{ei}$ (Delivery)
Objective 2: $Z_{max} = \sum_e \sum_i A_{ei} * X_{ei}$ (Quality)
Objective 3: $Z_{min} = \sum_e \sum_i C_{ei} * X_{ei}$ (Cost)

Subject to:

$$\sum_j X_{ei} \leq D_e , \forall e$$

$Q_{ijmin} * Y_{ei} \leq X_{ei}, \forall_e; Q_{ijmax} * Y_{ei} \geq X_{ei}, \forall_e; \sum_i Y_{ei} \geq n_e, \ni e; P_{ei}X_{ei} \leq U_i$
$\sum_e \sum_i X_{ei} = \text{Total number of products}; \sum_e \sum_i C_{ei}X_{ei} = Total\ Costs;$
$X_{ei} \geq 0 \ \&\ integer, Y_{ei} = 0\ or\ 1 ; Y_{ei} = 0 ; \forall_{(e,i)} \in no\ supplied$

A machining company desires to select appropriate supplier to purchase 4 product materials. The company has three suppliers (A1, A2, and A3), three decision makers (D1, D2, D3) in the committee. Then, the criteria for consideration are Delivery (C1), Quality (C2) and Cost (C3). In this problem, the demand is predicted to be around 1,300 units.

These three decision makers used the linguistic variables as shown in Table 1 to access the importance of criteria and demand constraint. The linguistic values determined by decision makers are shown in Table 2.

Using the weights of each criterion and fuzzy constraint are calculated by using Fuzzy TOPSIS. Then, the closeness coefficients and final weights can be seen in Table 3. Characteristics of Delivery, Quality, Cost and Demand for each product constraints of each candidate supplier, (Supplier 1, 2 and 3) are presented in Table 4 and the data set for membership function can be calculated and shown in Table 5. Table 6 shows the minimum and maximum order quantity for each supplier and each product. Each supplier also imposes a purchasing budget for the company. This is maximum allowed budget that the company can spend on its products from each supplier

Table 1: Linguistic variables for rating

Linguistic Variables	Triangular fuzzy number
Very low (VL)	(1,1,2)
Low(L)	(1,2,3)
Medium Low (ML)	(2,3.5,5)
Fair (F)	(4,5,6)
Medium good (MG)	(5,6.5,8)
Good (G)	(7,8,9)
Very Good (VG)	(8,10,10)

Table 2: Importance weight of criteria from three decision makers

	D1	D2	D3
Delivery (C1)	VG	VG	G
Quality (C2)	G	G	G
Cost (C3)	G	MG	G
Demand	G	MG	MG

Table 3: Weights, distances and coefficients of each criterion and constraint

d^*	d^-	d^*+d^-	CC_i	Final weight
0.79	0.06	0.85	0.929412	0.275831
0.74	0.13	0.85	0.847059	0.251391
0.68	0.17	0.86	0.802326	0.238115
0.67	0.18	0.86	0.790698	0.234664

Table 4: Suppliers' quantitative information

Delivery (%)				
	Product 1	Product 2	Product 3	Product 4
Supplier 1	0.80	0	0.90	0.80
Supplier 2	0.75	0.85	0	0.85
Supplier 3	0.70	0.75	0.85	0.75
Quality (%)				
Supplier 1	0.8	0	0.75	0.95
Supplier 2	0.75	0.70	0	0.8
Supplier 3	0.70	0.85	0.8	0.7
Cost ($)				
Supplier 1	20	0	25	20
Supplier 2	25	30	0	25
Supplier 3	15	20	35	25
Demand for each product (units)				
	700	600	300	500

Table 5: The data set for the membership function

Criteria & constraint	$\mu = 0$	$\mu = 1$	$\mu = 0$
Delivery	1,028.7	1,093.7	-
Quality	1,002.2	1,093.7	-
Cost	-	34,165	29,850
Demand	1,200	1,300	1,500

Table 6: Minimum and maximum order quantity from Supplier *j* for raw material *i*.

Minimum order quantity (units)				
	Product 1	Product 2	Product 3	Product 4
Supplier 1	50	0	50	50
Supplier 2	50	50	0	50
Supplier 3	100	25	100	100
Maximum order quantity (units)				
Supplier 1	200	0	200	250
Supplier 2	350	450	0	200
Supplier 3	200	150	450	350

The multi-objective linear formulation of numerical example is presented. The objectives are to maximize Z_1 and Z_2 while minimize Z_3

$Z_1 = 0.8X_{1,1} + 0.75X_{1,2} + 0.70\,X_{1,3} + 0.85X_{2,2} + 0.75X_{2,3} + 0.90X_{3,1} + 0.80X_{3,2} + 0.85X_{3,3} + 0.80X_{4,1} + 0.85X_{4,2} + 0.75X_{4,3}$

$Z_2 = 0.8X_{1,1} + 0.75X_{1,2} + 0.70\,X_{1,3} + 0.70X_{2,2} + 0.85X_{2,3} + 0.75X_{3,1} + 0.90X_{3,2} + 0.8X_{3,3} + 0.95X_{4,1} + 0.8X_{4,2} + 0.7X_{4,3}$

$Z_3 = 20X_{1,1} + 25X_{1,2} + 15\,X_{1,3} + 30X_{2,2} + 20X_{2,3} + 25X_{3,1} + 30X_{3,2} + 35X_{3,3} + 20X_{4,1} + 25X_{4,2} + 25X_{4,3}$

s.t.:

$X_{1,1} + X_{1,2} + X_{1,3} + X_{2,2} + X_{2,3} + X_{3,1} + X_{3,2} + X_{3,3} + X_{4,1} + X_{4,2} + X_{4,3} = 1300$; $X_i \geq 0$, $I = 1, 2, 3$.

$$\mu_{z1}(x) = \begin{cases} 1 & for \quad Z_1 \geq 1{,}093.7, \\ (Z_1 - 1{,}028.7)/(65) & for\ 1{,}028.7 \leq Z_1 \leq 1{,}093.7, \\ 0 & for \quad Z_1 \leq 1{,}028.7 \end{cases}$$

$$\mu_{z2}(x) = \begin{cases} 1 & for\ Z_2 \geq 1{,}093.7 \\ (Z_2 - 1{,}002.2)/(91.5) & for\ 1{,}002.2 \leq Z_2 \leq 1{,}093.7, \\ 0 & for\ Z_2 \leq 1{,}002.2 \end{cases}$$

$$\mu_{z3}(x) = \begin{cases} 1 & for \quad Z_3 \leq 29{,}850, \\ (34{,}165 - Z_3/(4315) & for\ 29{,}850(x) \leq Z_3 \leq 34{,}165, \\ 0 & for\ Z_3 \geq 34{,}875 \end{cases}$$

$$\mu_{gd}(x) = \begin{cases} \dfrac{d(x) - 1200}{100} & for\ 1{,}200 \le d(x) \le 1{,}300, \\ \dfrac{1500 - d(x)}{200} & 1{,}300 \le d(x) \le 1{,}500\ , \\ 0 & for\ d(x) \le 1{,}200, \quad d(x) \ge 1{,}500. \end{cases}$$

From Table 2, the weight of delivery, quality and cost as well as the weight of fuzzy constraint were obtained though TOPSIS. It was found that w_1 = 0.276, w_2 = 0.251, w_3 = 0.238 and β_1 = 0.23.

Applying the membership function and the final weights, we can obtain :

Max $0.276\lambda_1 + 0.251\,\lambda_2 + 0.238\,\lambda_3 + 0.23\,\gamma_1$

s.t. :

$\lambda_1 \le ((0.8X_{1,1} + 0.75X_{1,2} + 0.70\,X_{1,3} + 0.70X_{2,2} + 0.85X_{2,3} + 0.75X_{3,1} + 0.90X_{3,2} + 0.8X_{3,3} + 0.95X_{4,1} + 0.8X_{4,2} + 0.7X_{4,3}\ ) - 1{,}028.7 / \ 65)$

$\lambda_2 \le ((0.8X_{1,1} + 0.75X_{1,2} + 0.70\,X_{1,3} + 0.70X_{2,2} + 0.85X_{2,3} + 0.75X_{3,1} + 0.90X_{3,2} + 0.8X_{3,3} + 0.95X_{4,1} + 0.8X_{4,2} + 0.7X_{4,3}\ ) - 1{,}002.2\ /\ 191.5)$

$\lambda_3 \le (34{,}165 - \ (20X_{1,1} + 25X_{1,2} + 15\,X_{1,3} + 30X_{2,2} + 20X_{2,3} + 25X_{3,1} + 30X_{3,2} + 35X_{3,3} + 20X_{4,1} + 25X_{4,2} + 25X_{4,3}) / 4315\ )$

$\gamma_1 \le 1500 - (X_{1,1} + X_{1,2} + X_{1,3} + X_{2,2} + X_{2,3} + X_{3,1} + X_{3,3} + X_{4,1} + X_{4,2} + X_{4,3})/200$

$\gamma_1 \le (X_{1,1} + X_{1,2} + X_{1,3} + X_{2,2} + X_{2,3} + X_{3,1} + X_{3,3} + X_{4,1} + X_{4,2} + X_{4,3}) - 1200/ 100$

$$\left.\begin{array}{l} X_{1,1} \le 200;\ \ X_{1,2} \le 350;\ X_{1,3} \le 200;\ X_{2,2} \le 450; \\ X_{2,3} \le 150;\ X_{3,1} \le 200,\ X_{3,2} \le 350;\ X_{3,3} \le 450; \\ X_{4,1} \le 150;\ X_{4,2} \le 200;\ X_{4,3} \le 350 \end{array}\right\} \text{Limit capacity of each supplier for each supplier}$$

$$\left.\begin{array}{l} 20X_{1,1} + 25\,X_{3,1} + 20\,X_{4,1} \le 10000 \\ 25X_{1,2} + 30X_{2,2} + 30X_{3,2} + 25X_{4,2} \le 12500 \\ 15X_{1,3} + 20\,X_{2,3} + 35X_{3,3} + X_4 \end{array}\right\} \text{Limit allowed budget for each supplier}$$

This problem was solved by using Microsoft Excel Solver. The optimal solution for the model can be presented in the Table 7.

Table 7: Recommended results of the model

Decision variables	Solution values (units)	
$X_{1,1}$	150	Product 1
$X_{1,3}$	200	
$X_{2,2}$	140	Product 2
$X_{2,3}$	150	
$X_{3,1}$	80	Product 3
$X_{3,3}$	100	
$X_{4,1}$	250	Product 4
$X_{4,2}$	200	
$X_{4,3}$	100	

Note: Z_1 = 1,093.3, Z_2 = 1,086.7, Z_3 = 3,1945

As seen in Table 7, the results of the model indicate that Product 1 should be purchased in the number of 150 units from Supplier 1 and 200 units from Supplier 2. Product 2 should be purchased in the number of 140 units from Supplier 2 and 150 units from Supplier 3. Product 3 should be purchased in the number of 80 units from Supplier 1 and 100 units from Supplier 3. Product4 should be purchased in the number of 250 units from Supplier 1, 150 units from Supplier 2 and another 100 units from Supplier 3.

4. CONCLUSIONS

Even though, certain types of raw materials/products purchased from different suppliers have been involved in these above mentioned studies, a certain degree of fuzziness and uncertainties has not yet been introduced into the consideration. This study focuses on fuzzy multi-objective linear model to deal with the problem. In this paper, a new model is developed that complements the weakness mentioned above and proposes a complete fuzzy multi-objective linear model approach for the supplier selection problem. In our proposed model, firstly a fuzzy supplier selection model with multiple products/suppliers, fuzzy objective functions (goals), fuzzy constraints and fuzzy coefficients is developed and then the developed model is converted to a single objective one step by step. The weights for selection criteria can be treated as equal or unequal importance according to DM’s preference. With the option of different weights, linguistic values expressed as trapezoidal fuzzy numbers are used to assess the weights of the factors. Similar to AHP or TOPSIS approaches, new terms are presented as Fuzzy Positive Ideal Rating (FPIR) and Fuzzy Negative Ideal Rating (FNIR) to compute weights of factors. Then applying suppliers’ constraints, goals and weights of the factors, a fuzzy multi-objective linear model is developed to overcome the supplier selection problem and assign optimum order quantities for each supplier in every product.

ACKNOWLEDGEMENT

This study was supported by the research grant from Bangchak Petroleum Public Company Limited. The authors are grateful for this financial support.

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