A LOCATION-ROUTING PROBLEM
— THE CASE OF DISTRIBUTION CENTER IN THE US —

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Abstract

The goal of this study was to improve the warehouse and transportation management of a case study company which produced drink in the US. Currently, there is only one warehouse that covers all the demand in the US. Thus, transportation cost is very expensive and could not be responsive to the demand. Therefore, the objective of this study is to reduce transportation cost by selecting the new warehouse with the truck delivery routing. This problem becomes the Location Routing Problem (LRP). Then, we propose a mixed integer programming to minimize the total cost of fixed and variable cost of warehouse, vehicles and transportation cost. In addition, in order to make it computational feasible, we propose a solution method to make the possible zoning and select the warehouse with the truck routing within the zone. Then, we used IBM ILOG CPLEX to solve on the numerical case data to obtain the optimal result. The result showed that after selecting the having the new warehouses, the total weekly logistics cost was reduced by 28.7%.

Keywords: Vehicle Routing, Location Routing Problem, Logistics, Mixed Integer Programming

1. INTRODUCTION

A logistic system covers the entire process of moving raw materials and input requirements from suppliers to plants, the conversion of the inputs into products at certain plants, the movement of the products to various warehouses or depots, and the eventual delivery of these products to final customers. Major distribution costs normally consist of operating vehicles and crews' salaries. Consequently, a small saving in the distribution costs could result in a substantial total savings for an organization. This paper addresses a deterministic LRP that combines location problem (LP), the vehicle routing problem (VRP). It normally deals with the distribution from multiple depots to many customers whose demands are known with certainty and are to be served by multiple vehicles routing from each depot. Location Routing Problem (LRP) represents the interdependence between a strategic decision

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and a shorter-term tactical decision, by determining the number of depots, selecting the locations of depots, and configuring the routes that emanate from the selected depots. The objective is to minimize total costs that may include fixed costs of opening the depots and distribution costs incurred by delivery or pickup operations. The number of research literature in LRP is found to be limited when compared with the LP and VRP.

Although the location of depots is an infrequent decision to make when compared with vehicle routing, from a managerial viewpoint, simultaneously optimizing both decisions is quite important and worth consideration, since the decisions of opening depots and selecting locations of depots can have significant effect on how the routing operates and the amount of costs incurred. Specifically, the locations of depots influence the decision of customer allocation to the selected depots and, subsequently, the sequence of customers included in each route as well as the number and length of routes starting from and ending at each depot. Such an integrated design problem can enable the organization to achieve higher productivity and cost savings in the distribution operations.

Since both subproblems of LRP, namely LP and VRP, belong to the NP-hard class of optimization problems (Lenstra and Rinnooy Kan, 1981), LRP is inevitably NP-hard, and, therefore, no polynomial-bound exact algorithm has yet been found. The remainder of the paper is organized as follows. Section 2 gives a literature review. Section 3 describes the modified LRP model.

2. LITERATURE REVIEW

Although LRP has been developed and studied only during the past four decades, a few extensive surveys can be already found in the literature, such as Madsen (1981), Laporte (1988), and Min et al. (1998). Min et al. (1998), the most recent review, classified LRP into two categories based on the problem perspective and solution methods. The problem perspective includes for example characteristics of facilities and vehicles, the nature of demand and supply (deterministic or stochastic), hierarchical planning levels, and planning horizon, while the solution methods include naturally exact and heuristic algorithms. Exact algorithms including branch and bound, integer programming, and dynamic programming can be found in many hazardous waste applications such as Revelle et al. (1991) and List and Mirchandani (1991). Most heuristics found in the literature, on the other hand, apply some combinations of the four strategies, namely location-allocation-first, route-second; route-first, location-allocation-second; savings/insertion; and tour improvement/exchange. A few examples of those papers applying these concepts are mentioned.

Considering LRP in the newspaper distribution application, Jacobson and Madsen (1980) proposed two heuristics applying the location-allocation-first, route-second and the savings method) to solve a two-level routing-location problem, which was later solved by Madsen (1983) by three heuristics, namely the tree-tour heuristic; the location-allocation-first, route-second; and the SAV-DROP heuristic. Perl and Daskin (1985) formulated a mixed-integer program for the warehouse location-routing problem (WLRP) to determine the number, sizes, and locations of the warehouses to be established, the allocation of customers to warehouses, and the delivery routes, so as to minimize the sum of transportation costs and warehousing costs. A heuristic algorithm was proposed using rather the concept of route-first, location-allocation-second.

3. PROBLEM DESCRIPTION AND MODEL FORMULATION

According to Min et al. (1998), our problem can be characterized as LRP with a single stage, deterministic, multiple facilities, multiple vehicles, capacitated vehicles, and capacitated facilities. We assume in the model that the delivery fleet is homogeneous, consisting of standard vehicles, and that the number and capacity of vehicles are known. Given that N, M, and K are the numbers of customers, depot or distribution center (DC) sites, and vehicles, respectively, we present our notation and model, by modifying that of Perl and Daskin (1985) as follows:

Indices:

i = customer index ($1 <= i <= N$)

j = DC site, $N+1 \leq j \leq N+M$

k = route or vehicle, $1 \leq k \leq K$

h, g = any node, either customer or DC, $1 \leq h, g \leq N+M$

Parameters:

D_{ij} = distance between points i and j

Q_i = demand or requirement of customer i

F_j = fixed cost of establishing DC j

V_j = variable cost per unit throughput at DC j

T_j = maximum throughput at DC j

C_k = capacity of vehicle (or route) k

O_k = fixed cost of a vehicle (constant)

P_k = variable cost per distance unit of vehicle or route k

Decision Variables:

$$x_{ghk} = \begin{cases} 1 & \text{if point } g \text{ precedes } h \text{ on route } k \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ij} = \begin{cases} 1 & \text{if customer } i \text{ is allocated to DC } j \\ 0 & \text{otherwise} \end{cases}$$

zj $= \begin{cases} 1 & \text{if DC is opened at site j} \\ 0 & \text{otherwise} \end{cases}$

LRP-MIP Model:

$$\text{Min} \quad \sum_{j=N+1}^{N+M} F_j \cdot z_j + \sum_{j=N+1}^{N+M} V_j \left(\sum_{i=1}^{N} Q_i y_{ij} \right) + \sum_{k=1}^{K} \sum_{g=1}^{N+M} \sum_{h=1}^{N+M} M_k \cdot D_{gh} \cdot x_{ghk} + O \sum_{k=1}^{K} \sum_{g=N+1}^{N+M} \sum_{h=1}^{N} \cdot x_{ghk} \qquad (0)$$

Subject to

$$\sum_{k=1}^{K} \sum_{h=1}^{N+M} x_{ihk} = 1 \qquad \forall i, h \qquad (1)$$

$$\sum_{i=1}^{N} Q_i \sum_{h=1}^{N+M} x_{ihk} \leq C_k \qquad \forall k \qquad (2)$$

$$a_i - a_{i'} + N x_{i,i',k} \leq N - 1 \qquad \forall i \neq i', k \qquad (3)$$

$$\sum_{g=1}^{N+M} x_{hgk} - \sum_{g=1}^{N+M} x_{ghk} = 0 \qquad \forall k, h \qquad (4)$$

$$\sum_{i=1}^{N} Q_i y_{ij} - T_j z_j \leq 0 \qquad \forall j \qquad (5)$$

$$\sum_{i=1}^{N+M} x_{hik} - \sum_{h=1}^{N+M} x_{jhk} - y_{ij} \leq 1 \qquad \forall i, j, k \qquad (6)$$

$$x_{ghk} \quad \text{binary} \qquad \forall g, h, k$$

$$y_{ij} \quad \text{binary} \qquad \forall i, j$$

$$z_j \quad \text{binary} \qquad \forall j$$

$$a_i \geq 0 \qquad \forall i$$

The objective function is to minimize the sum of fixed warehousing cost, variable warehousing cost, delivery cost as a function of distance, and fixed variable cost. Constraint (1) requires that each from-to customer must be served by exactly one route, assuming that demand requirement of each customer must be less than the vehicle's capacity. Constraint (2) assures that delivery demand assigned to a vehicle must not exceed the specified vehicle capacity. Constraint (3) is to eliminate subtours, while Constraint (4) guarantees that each customer point entered by a vehicle be left by the same vehicle. Constraint (5) requires that the customer demand served by a particular DC not exceed the maximum throughput of that DC. Constraint (6) assures that if a customer is served by a vehicle of a particular DC, the customer is assigned to that DC, linking the allocation and routing components. Constraints

(7) – (9) specify the binary type of all decision variables, while constraint (10) requires nonnegativity of a_i, associated with the subtour elimination constraint.

To verify the above model and to establish a basis for comparison on runtime and solution quality, some small LRP examples were randomly created and solved by an optimization package, IBM ILOG CPLEX version 12.4.

4. CASE STUDY ANALYSIS

The drink company who produces and distribute in the United States currently has only one warehouse located in California that distributes all over 48 states. The weekly renting cost for a warehouse is $15,000 and the transportation cost is $0.85/km. Moreover, the company need to rent the truck where the renting cost varied by its capacity. The current weekly transportation cost is $128,859.15 which accounted for 89.6% for the total weekly logistics cost. The company, therefore, considering to set up additional warehouses to reduce the total logistics cost.

The solution process for the LRP is divided into the following three steps:

1. In the first step, the potential zones for warehouse are determined by geography.
2. In the second step, the two possible warehouse locations are determined on the basis of the two highest demands in the zone.
3. In the third step, the warehouse location and vehicle routing are optimally determined by the proposed LRP-MIP model.

Table 1: The list of states in each divided zone.

Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6	Zone 7
Massachusetts	New York	Florida	Illinois	Texas	Arizona	Washington
Connecticut	Indiana	Virginia	Minnesota	Tennessee	Colorado	Oregon
Maine	New Jersey	North Carolina	Missouri	Louisiana	Utah	Nevada
New Hampshire	Pennsylvania	Washington DC	Nebraska	Oklahoma	Idaho	California
Rhode Island	Ohio	Maryland	Kansas	Alabama	New Mexico	
Vermont	Michigan	Georgia	Iowa	Arkansas	Montana	
		South Carolina	North Dakota	Mississippi	Wyoming	
		Kentucky	South Dakota			
		Delaware	Wisconsin			

Due to the computational limitation on the VRP-MIP model, the number of state in each zone cannot be more than 12 states. Therefore, we divide into 7 zones with all the states shown in Table 1. The parameter for the number of vehicle, the variable distance cost, the warehouse renting cost, and total weekly demand are shown in Table 2. The detailed weekly demand for each state and the distance matrix for each pair are not presented here.

Table 2: The parameter data for each studied zone.

	Zone						
	1	2	3	4	5	6	7
Number of customer (N)	6	6	9	9	7	7	4
Number of depot (M)	2	2	2	2	2	2	2
Number of vehicle (K)	4	4	4	4	4	4	4
Capacity of vehicle (C_k)	500	1200	800	400	400	400	2500
Fixed cost of a vehicle (O_k)	50	120	80	40	40	40	250
Variable cost per distance unit of vehicle (P_k)	0.85	0.85	0.85	0.85	0.85	0.85	0.85
Fixed cost of warehouse (F_j)	7,500	7,500	7,500	7,500	7,500	7,500	7,500
Variable cost of warehouse (V_j)	$1	$1	$1	$1	$1	$1	$1
Warehouse capacity (T_j)	6,000	6,000	6,000	6,000	6,000	6,000	6,000
Weekly demand	601	1,323	2,131	471	539	448	3,406

After we have all seven zones, we select the two states with the highest demand for the drink. They are the possible locations for the warehouse in each zone as shown in Table 3.

Table 3: The list of possible warehouse location in each zone and its demand.

Zone	State	Demand/week	State	Demand/week
1	Massachusetts	341	Connecticut	150
2	New York	1002	Indiana	93
3	Florida	798	Virginia	379
4	Illinois	294	Minnesota	66
5	Texas	360	Tennessee	41
6	Arizona	214	Colorado	130
7	California	2406	Washington	472

5. RESULT AND DISCUSSION

We implement LRP-MIP model with the IBM ILOG CPLEX version 12.4 and solve all problem on the computer with the CPU Intel Core 2 Duo Processor P8600 (2.40 GHz), memory 3 GB, on Windows Vista Business.

Table 4 summarizes all of the computational result for our case study. The computational times for all zones are within acceptable time which is less than 3 minutes. The computational times for zone 3 and 4 are higher because they contain the largest number of customers. The numbers of vehicles used in all zones, except zone 3, are two. There are four vehicles in zone 3 because the demand is much higher than the capacity of the vehicles.

The detailed routings for each zone are shown in Figures 1-3. Since, the routings are arranged with the consideration of the vehicle capacity only, the distance for the routes are not well balance. In most cases, there will be a long routing and a very short routing. For those long routings, the driving could be longer than a week and, as a result, might not be able to make a weekly deliver schedule.

Table 4: The computational result for each case.

Zone	Time (sec.)	Total Cost ($)	Warehouse Location	Number of vehicles
1	6.04	9,368.05	Connecticut	2
2	1.45	11,344.40	New York	2
3	20.94	20,271.30	Virginia	4
4	152.23	10,983.50	Illinois	2
5	4.95	11,521.55	Texas	2
6	1.84	16,120.1	Colorado	2
7	1.09	20,410.6	California	2

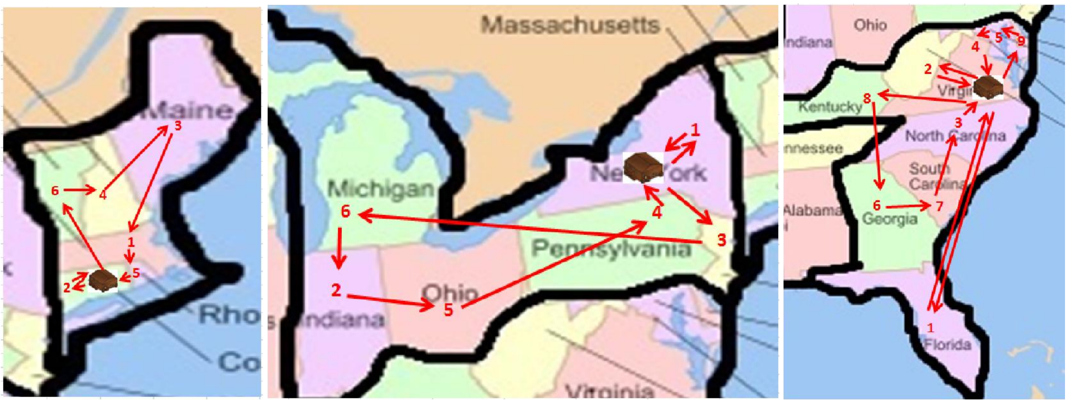

Figure 1: Possible warehouse location and the routing for zone 1, 2 and 3.

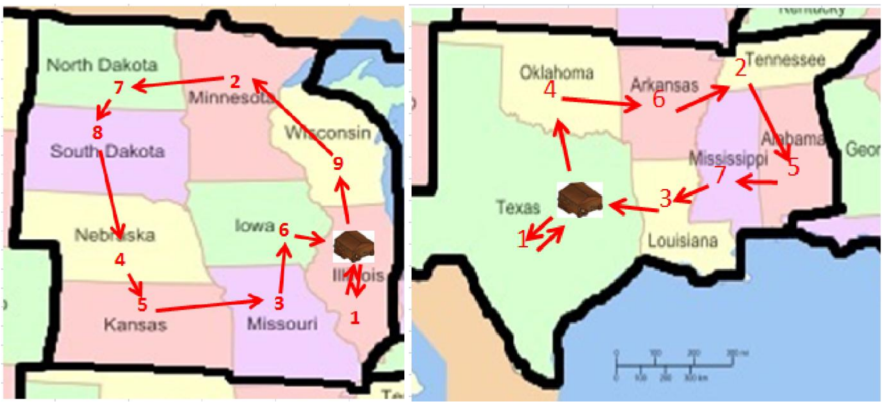

Figure 2: Possible warehouse location and the routing for zone 4 and 5.

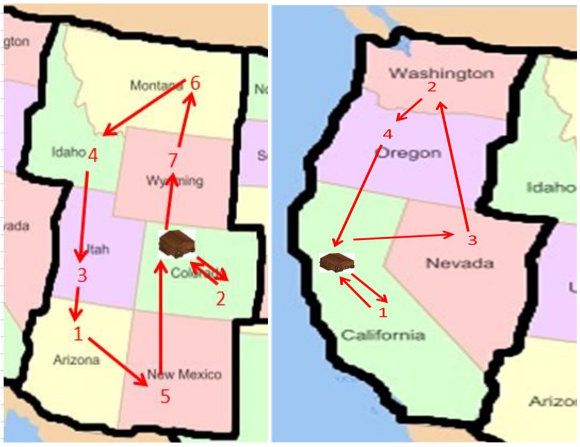

Figure 3: Possible warehouse location and the routing for zone 6 and 7.

Table 5 shows the cost comparison between the current strategy and the proposed strategy. For the current route planning the transportation cost is almost 90% of total logistics cost. However, the proposed strategy could reduce the transportation cost dramatically by 52.33% and reduce the total logistics cost by 28.73%.

Table 5: Cost comparison between the current and proposed strategy.

Cost	**Current Strategy**		**Proposed Strategy**	
	Weekly Cost ($)	%	Weekly Cost ($)	%
Transportation Cost	$ 128,859.15	89.6	$ 61,419.00	59.9
Warehouse Cost	$ 15,000.00	10.4	$ 41,111.15	40.1
Total Logistics Cost	$ 143,859.15	100.0	$ 102,530.15	100.0
% Change			- 28.73%	

6. CONCLUTION

The presented LRP is to solve a method for combining location planning and vehicle routing. We propose the solution technique for solving large scale location and routing problem. The technique is to reduce the size of the problem by zoning and then solve for the LRP for each zone. The case study has shown that the proposed technique can lead to significant savings in logistics costs. Nevertheless, reflecting a realistic distribution of goods within the location planning process can provide the opportunity to obtain good solutions. For further study, we recommend to restrict the distance for each vehicle for the reasonable driving time.

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