The Application of Polychromatic Set Theory in the Assembly Sequencing of Blocks for Offshore Rigs

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Abstract

The objective of this research is to improve the construction process of off-shore rigs blocks by means of the automated identification of sub-assemblies together with their assembly sequence based on the use of Polychromatic Set Theory. Polychromatic set theory (PST) was developed by V. V. Pavlov, the Russian aeronautical designer. The theory, in its final form, was presented in 2002 and has been used in a number of engineering areas, particularly in Russia. It has been applied to a wide variety of industrial problems. The theory has found to be useful in such diverse areas as automating sequences and sub-assembly detection in automobile bodies and assembly planning, simulating complex objects and systems, and building disassembly models. PST has been used in production areas, rather than construction areas. One area that it has not been applied to is in the construction of blocks for off-shore rigs. The construction of blocks for off-shore rigs is a complex task involving many stages, from the engineering design, detailed design, the cutting of parts, the construction of sub-blocks¹, and the construction of sub-assemblies right through to the final erection of the rig.

Keywords: Polychromatic Sets, Off-shore Rigs, Assembly Sequencing, Block Construction

1. INTRODUCTION

The assembly sequencing of off-shore drilling rig blocks is the essential first step in minimizing the construction makespan for the whole rig. This paper will focus on using PST for automatically identifying the assembly sequence of sub-blocks/sub-assemblies of

¹ Sub-block is the term used in some shipyards and it refers to a slightly more complex part that is formed when two or more parts are welded together. Two or more sub-blocks are combined to form a sub-assembly. A sub-assembly may also be formed by more than two sub-assemblies.

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off-shore rig blocks. It begins with a brief survey of various assembly methods, following which a definition of PST is given. A case study using PST is then given, showing its power and flexibility.

Experience is the method used by design engineers to identifying sub-blocks for off-shore rig blocks. Ship's design software allows for manual identification of part groups. The combination of sub-blocks into sub-assemblies is left to the discretion of the production engineer.

An off-shore rig is constructed from hundreds of blocks. Generally, no two blocks have exactly the same sub-assembly configuration; hence, any lessons learned from one block in minimizing its makespan will not apply to any other block. Each block has its own unique assembly sequence of sub-assemblies. Although none of the research from a production environment can be applied directly to off-shore rigs, lessons can be taken from them and applied to a construction environment. PST has been used in the production environment and has proved successful. PST has shown great flexibility coupled with mathematical rigour. These factors alone recommend it for using in the assembly sequencing of off-shore rig blocks.

The luxury of time does not apply to a construction environment; early completions have benefits and late completions attract penalties. Working with one-off construction sequences means that there is no history from which to learn. The number of parts in a block and the possible connective combinations is extremely large. Approaches that use combinatorial methods have been rejected for this reason. The reasons for rejection of other methods are given in the literature review. PST, with its strong mathematical basis provides a rapid method that does not require extensive computational resources, as would a GA for example or a graph search.

2. LITERATURE REVIEW

Assembly sequencing is fundamental in identifying the construction sequence of any sub-assembly. Proper planning of an assembly of a block for an off-shore rig is essential to any automatic or computer based generation of an assembly sequence (Bai et al, 2005). The generation of an assembly sequence for any product can be quite complex. Several methods have been proposed in an attempt to reduce this complexity. These methods may be broadly classified into the following categories: dis-assembly based reasoning, knowledge-based reasoning, genetic algorithms and neural networks, precedence relations, miscellaneous methods and polychromatic sets.

Dis-assembly based or geometric reasoning comes in various forms. Dini & Santochi (1992), use three types of matrices: interference, contact and connection to derive the dis-assembly sequence. Whereas Homem de Mello's (1991) algorithm uses the relational mode together with a geometric model of the assembly together with geometric reasoning to

produce a dis-assembly sequence. Lai & Huang (2004), generate an assembly sequence using assembly precedence relations created by an engineer using computer based liaison graphs and matrices. Jin et al (2010), generate a dis-assembly interference matrix which is based on the model's component interference matrix. This is input to an ant colony algorithm for the purposes of optimization.

Knowledge-based reasoning has also received the attention of researchers. Chakrabarty & Wolton (1997), propose that the assembly sequence planner uses a structured hierarchy both as a framework for structured-dependent definition of a *good plan*, and also a tool for locating *good plans* quickly by using high-level expert advice. Dong et al (2007), propose a method that uses both the geometric and non-geometric knowledge and produced sequences that required less computation time and led to sequences that are more practical.

Genetic algorithms and neural networks are widely used in NP-hard problems. Bonneville et al (1995) use a GA that generates and evaluates assembly plans. The initial population is the set of valid assembly plans produced by an expert of the product in question. This approach rapidly generates a set of good assembly plans. Chen & Liu (2001), not relying upon experts, propose *an adaptive genetic algorithm* to locate global-optimal or near global optimal sequence. Sinanoglu & Börklü (2005) use a neural network approach to develop assembly sequence plans.

Bourjault's (1984) seminal work uses precedence relations to construct an assembly tree. His work was refined by De Fazio & Whitney (1987) and later by Wilson (1995) Minzu et al (1999) obtains feasible assembly sequences by merging assembly sequences. Graphical methods are the main stay of many of the methods already mentioned. Herioud et al (2002) extends Miznu et al (1999) by using hypergraphs for assembly system design. Lee & Gossard (1985) use a hierarchical data structure to represent assemblies in conjunction with a database that holds topological and geometric information and is supported by a data structure that identifies how all components in an assembly are connected.

Gou et al (2010) use a hierarchical constraint assembly model for assembly sequence planning where a model is composed into small constructible components; this process is recursive, with constraint being formed between the 'layers'. Lai & Huang (2004) use liaison and precedence matrices along with precedence relations to obtain the optimized assembly sequence.

Petri nets, with an algorithm, is the method Zhang (1989) uses to generate assembly plans. In a similar vein, Lu et al (1993) uses the Hidag tree to represent assembly sequences which can be translated into a set of linear sequences.

All these methods mentioned have some weakness. Dis-assembly sequencing assumes that the assembly is the reverse of the dis-assembly, which is not always the case. Knowledge base systems require expert input, and this, at best, is subjective knowledge. GAs do not guarantee a global solution, and are certainly not suitable for ad-hoc work. Precedence relationships can be complex and convoluted to use.

PST has been receiving some attention and has been applied to a wide variety of industrial type problems. PST is used in simulation modelling, product life cycle simulation, product conceptual design, concurrent engineering and process modelling (Li & Xu (2003)). Li et al (2006) apply PST to product assembly modelling, work flow modelling and tolerance modelling. Gao & Li (2006) apply PST to the conceptual design in a product development. PST is widely used in the Russian aviation industry and astronavigation (Zhao & Li, 2006, 2009). Zhao et al (2006) use PST for car body assembly. Liu et al (2010) use PST in the process model for body-in-white welding assembly. They analyse the connection and interference relationship with an algorithm to generate a welding assembly sequence. Wang & Li (2012) have applied PST to scalable routing modelling for wireless ad hoc networks as well as spectrum access in cognitive radios. This brief survey of the literature for PST shows it flexibility and diverse use. PST does not require any specialised knowledge per se, it is straight forward to use and it is easily extendible.

3. POLYCHROMATIC SET THEORY

3.1 Polychromatic Sets (PS)

In conventional set theory, a set *A* is defined as $A = (a_1, ..., a_j, ..., a_n)$. For elements, $a_i, a_j \in A$, the difference is in name only, even though each element could be different. With conventional set theory, it is not possible to represent all other characteristics or properties. In polychromatic sets, as its name suggests, each element has pigmentation as does the entire set, or any subset. Different colours are used to represent the research object as well as properties of its elements. PST defines a colour set by *F*, where for any element $a_i \in A$, the colouring of the element is given by equation (1). The colour set $F(a_i)$ corresponds to every element in $a_i \in A$. In the same manner, the colouring of the whole set is given by equation (2). The colour set F(A) corresponds to the entirety of *A*.

$$F(a_i) = \left\{ f_1(a_i), \cdots f_j(a_i), \cdots f_m(a_i) \right\} = \left\{ f_1, \cdots f_j, \cdots f_m \right\}$$
(1)

$$F(A) = \{F_1(A), \cdots F_j(A), \cdots F_m(A)\} = \{F_1, \cdots F_j, \cdots F_m\}$$
(2)

 $F_j(A)$ and $F_j(a_i)$ are identified as Polychromatic sets unified colours and individual colour elements a_i respectively. When an object of investigation is represented in terms of polychromatic sets, its colours $F_j(A)$ and $F_j(a_i)$ the colour of element a_i , corresponds to the j^{th} character of the object or the element.

3.2 Individual Pigmentation

In Boolean matrix form, the individual pigmentation $A \times F(a)$ of all the element of polychromatic sets can be represented as follows: -

$$\|C_{i(j)}\|_{A,F(a)} = [A \times F(a)] = \begin{pmatrix} F_1 & \cdots & F_j & \cdots & F_m \\ c_{1(1)} & \cdots & c_{1(j)} & \cdots & c_{1(m)} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ c_{1(i)} & \cdots & c_{i(j)} & \cdots & c_{i(m)} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ c_{n(1)} & \cdots & c_{n(j)} & \cdots & c_{n(m)} \end{pmatrix} a_n^{(1)}$$
(3)

Where $C_{i(j)} = 1$ if $F_j \in F(a_i)$ and $F(a) = \bigcup_{i=1}^n F(a_i)$

Here the colour F_j is represented by the logical variable $F_j = \begin{cases} 1. \text{ if } F_j \in F(a_i) \\ 0. \text{ if } F_j \notin F(a_i) \end{cases}$

3.3 Individual and Unified Colours

The key factor that determines the availability of a unified colour is the existence of individual colours of the element that constitutes a polychromatic set. By way of example, the existence of unified colour $F_j(A)$ is determined by the existence of individual colours with the same name as element a_i or a number of such elements.

 $\|C_{k(j)}\|_{F(a),F(A)} = [F(a) \times F(A)]$ where $C_{k(j)} = 1$, assuming that individual colours affects the

existence of the unified colour F_k . The relationship between the unified colour of polychromatic sets and the individual colour of element $a_i \in A$ can be represented by the following Boolean matrix:-

$$\left\|C_{i(j)}\right\|_{\mathbf{A},F(A)} = \left[A \times F(A)\right] \tag{4}$$

where $F_j = \begin{cases} 1. \text{ if } F_j \in F(a_i) \\ 0. \text{ if } F_j \notin F(a_i) \end{cases}$

The difference between (3) and (4) is that the inclusion of F(A) are only unified colours. A PS-set is represented by six components:-

$$PS = (A, F(a), F(A), [A \times F(a)], [A \times F(A)], [A \times A(F)])$$

It is not necessary to have all components in the PS-set for practical applications.

Offshore drilling rigs are composed of sections referred to as 'blocks'. A block can be of any shape, and any weight up to a lifting limit of the construction yard. A single block may weigh anything from tens of kilogrammes to well over 150 tonnes. Blocks are joined together to construct larger blocks or modules. The focus of this paper is the identification of sub-blocks, sub-assemblies and their assembly sequence.

A block is composed of possibly hundreds of parts. These parts are cut out by cutting machines and then assembled into what is often termed 'sub-blocks', which is just a part of a

block, or a sub-assembly. These sub-blocks can be joined together to make sub-assemblies and then they are assembled together to make a block. The assembly sequence is not the same as the final welding sequence. The assembly sequence usually moves longitudinally and transversely. The reason for this is that in the assembly process, parts are just tack welded into position.

Figure 1 shows the hierarchical relationship of the components of a block.



Figure 1: Block-Parts Relationship

In the construction company, the whole process of selection and identification of sub-blocks that can be combined into sub-assemblies is entirely manual. Unlike a production process, or even a construction process of housing estates where there is a high degree of repartition, the construction of a given block for off-shore rigs is probably a unique process, since it is common for no one block to be the same as another block.

The proposal here is to automatically identify the sub-blocks. Having identified the sub-blocks, sub-assemblies and assembly sequence can be obtained.

Figure 2 shows an example of a sub-assembly of consisting of several sub-blocks. Manually, an engineer would select a single panel with its associated stiffener as a sub-block. In Figure 2, there would be 8 sub-blocks, each plate being a sub-block. The decision of when the sub-blocks are combined into larger units is left to the discretion of the production engineer.



Figure 2: Final Assembly of Several Sub-Blocks into an Assembly

5. ASSEMBLY SEQUENCING

The work described here is a part of a much larger piece of research. The full research uses Just-in-Time (JIT) concepts in the production of off-shore rigs. It begins with the assembly sequence, from which the construction times for each sub-assembly can be calculated. The combination of sub-block/sub-assemblies is optimised so that the welding time, in the final position of the sub-blocks/sub-assemblies, is minimized. Knowing the construction time that

each sub-block/sub-assembly takes, the order of its assembly into its final position, the number of workers available, a makespan for the whole block construction can be calculated.

The full research covers, inter alia, the identification of generic constructions, such as truncated tetrahedral, boxes and polyhedral in general. It also identifies the welding position required, for the joining of sub-blocks. This is done in order to reduce the welding in the final assembly and to reduce the overhead and vertical welding in the final assembly. The focus of this paper is on the use of PST to identify sub-blocks/sub-assemblies and assembly sequences.

5.1 Basic Assembly Connection Matrices and Equations



Figure 3: Simple Illustration of Sub-block Sequencing

The first step is to calculate the bounding boxes of each sub-block. Connection identification uses bounding boxes. Figure 3 is a diagrammatic representation of the bounding boxes of some sub-blocks.

In PST, sub-blocks can be represented by the basic elements of the set $A\{a_1, a_2, ..., a_n\}$. The connection relations between sub-blocks can be represented by the colour set F(A). The Cartesian product $[A \times F(A)]$ gives the global contour as shown in (5). Here the set A is the Cartesian product of the set of sub-blocks. The set F(A) is the connection relationship. Where F_1 is connection in the +X direction, F_2 is connection in the – X direction. F_3 is connection is in +Y direction, F_4 is connection in the –Z direction. F_5 is connection in the +Z direction, F_6 is connection in the –Z direction.

In the matrix equation (5) $C_{i(j)} = \begin{cases} 1, F_j \in F(a_i) \\ 0, \text{ otherwise} \end{cases}$

The physical assembly sequence is shown in Figure 3. Starting at the relative (0, 0, 0)

location, the fitting sequence moves along the X-axis, until there are not more connected sub-blocks. The sequence then begins again with the sub-block that is adjacent and connected in the Y-direction.

The next step is to select the all parts that are connected in the +X direction. The sub-blocks in the first column of Figure 3 are easily identified as follows:-

$$C_{i(1)} = 1 \wedge (C_{i(3)} = 0 \wedge C_{i(4)} = 0 \wedge C_{i(5)} = 0 \wedge C_{i(6)})$$
(6)

The general connection equation for any part connected in the +X direction, at the lowest level is given by:-

$$C_{i(1)} = 1 \wedge (C_{i(5)} = 0 \wedge C_{i(6)} = 0)$$
(7)

A new set B is created from the connected pairs, (a_i, a_j)

 $B_1 = \left\{ b \mid C_{i(1)} = 1 \land (C_{i(5)} = 0 \land C_{i(6)} = 0) \right\}.$ PST is now applied to the new set, $B_1 \left\{ b_1, b_2, b_3, \dots b_k \right\}$ from which a new global contour is derived.

$$\|C_{i(j)}\|_{B_{1},F(B_{1})} = [B_{1} \times F(B_{1})] = \begin{bmatrix} b_{1} \\ (b_{1},b_{2}) \\ (b_{1},b_{3}) \\ (b_{1},b_{3}) \\ (b_{1},b_{3}) \\ \vdots \\ (b_{1},b_{3}) \\ \vdots \\ (b_{1},b_{n}) \\ \vdots \\ (b_{n-1},b_{n}) \end{bmatrix} = \begin{bmatrix} B_{1} \times F(B_{1}) \end{bmatrix} = \begin{bmatrix} b_{1} \\ (b_{1},b_{3}) \\ (b_{1},b_{3}) \\ \vdots \\ (b_{1},b_{2}) \\ (b_{1},b_{3}) \\ \vdots \\ (b_{1},b_{2}) \\ \vdots \\ (b_{2},b_{3}) \\ \vdots \\ (b_{2},b_{2}) \\ (b_{2},b_{3}) \\ \vdots \\ (b_{2},b_{3})$$

In the matrix equation (8) $C_{i(j)} = \begin{cases} 1, F_j \in F(a_i) \\ 0, \text{ otherwise} \end{cases}$

Using the new global contour for set B_1 , further connections in the +X direction are identified in the same way they were in the initial step of identification. This time, however, the connections are not between sub-blocks, but between sub-blocks, since two or more sub-blocks comprise a sub-assembly.

The set B_1 is process in exactly the same way as the previous set, using PST. From this, another set B_2 can be derived using the same Boolean logic. In this way, there is a reducing number of sub-blocks to deal with as they are progressively combined into larger sub-blocks.

Although all components in a block are connected (welded), this does not mean there is a continuous connection of sub-blocks from one end of a block to the other. These sub-blocks will not be carried over from one matrix to the next. They can be easily identified by the connection equation

$$C_{i(1)} = 0 \land (C_{i(5)} = 0 \land C_{i(6)} = 0)$$
(9)

Given that sub-block 1 in Figure 3 is identified as the 'starting sub-block', other sub-blocks and their assembly sequence follow by using PST. In order to identify the 'next starting block', the following connection matrix in (10) is used.

$$\|C_{i(j)}\|_{A,F(A)} = [A \times F(A)] = a_i \begin{bmatrix} c_{2(1)} & c_{2(2)} & c_{2(3)} & c_{2(4)} & c_{2(5)} & c_{2(6)} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \cdots \\ c_{i(1)} & c_{i(2)} & c_{i(3)} & c_{i(4)} & c_{i(5)} & c_{i(6)} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \cdots \\ c_{n(1)} & c_{n(2)} & c_{n(3)} & c_{n(4)} & c_{n(5)} & c_{n(6)} \end{bmatrix}$$

$$(10)$$

The connection matrix for all sub-blocks connected to sub-block a_1 is shown in (10).

To identify all sub-blocks that are connected to a_1 in the Y+ direction, equation (11) is used.

$$C_{i(3)} = 1 \wedge (C_{i(5)} = 0 \wedge C_{i(6)} = 0)$$
(11)

In the case of the example in Figure 3 is sub-block 4. Having identified the first sub-block connected to sub-block 1, in Figure 3 in the +Y direction, it is now possible to identify all connections in the +X direction starting from this sub-block (sub-block 4, in the example). The process of obtaining successive, but decreasing set of connection objects until a null set is obtained has been described above.

Equation (11) is used to identify the sub-block connected to sub-block 4, in our example, and the process of identifying all connections in the +X direction beginning at this block is the same as described previously, that is using connection equation (6)

Having completed the identification of all connections at the base level, the next step is to identify any sub-blocks at a higher level. Matrix (10) is used in this step. This time, however, the connection equation is given by

$$C_{i(1)} = 1 \wedge (C_{i(3)} = 0 \wedge C_{i(4)} = 0 \wedge C_{i(5)} = 1 \wedge C_{i6} = 0)$$
(12)

Starting from this sub-block, the process of identifying all connections at this level is the same as has been described for sub-blocks at the lowest level. When all sub-blocks and their connections at this level have been fully identified, the next level of connections in the +Z direction is process. This recursive terminates when there are no more levels to process.

5.2 Advanced Assembly Connection Matrices and Equations

Blocks for off-shore rigs may come in almost any combination of parts, shape and part-to-part

connection. Here we can show the power of using PST, when moving from the basic assembly connection in section 5.1, to a real-world complex study. To do this, it is necessary to deal with some block structures.



Figure 4: Full View of Double Bottom Block



Figure 6: Bulkhead with Stiffeners, Flanges and Plates



Figure 5: 'Inside' of Double Bottom Block



Figure 7: "Base" Plates with Stiffeners

Figure 4 shows a completed block that measures 9.1 x 11.3 meters. Figure 5 shows the 'inside' of the block. To make it easier to identify different parts, even though they are of the same type, (plates for example), the parts have been shaded differently. As can be seen in Figure 5, the block has both longitudinal and transverse sections. These sections are composed of sub-blocks. There is also a section that is on the block boundary. The block in Figure 4 is known as a 'double bottom' as there is both a 'top' and a 'bottom' of the block. Blocks that only have a 'bottom' are referred to an 'open hull'.

The steps involved in identifying the assembly sequence are as follows: -

- Calculate the bound box for each part.
- Identify the 'base' plate and its connecting parts
- Identify the 'base' plate and connections assembly sequence
- Identify the 'top' plate and its connecting parts.
- Identify the 'top' plate and connections assembly sequence.
- Identify the bulkheads
- Identify stiffeners/flanges/plates/bulkheads/ connected to the bulkheads
- Identify bulkhead sub-block fitting sequence
- Identify plates fitting sequence to bulkhead sub-blocks
- Identify bracket fitting sequence to bulkhead sub-blocks
- Fit 'top' section.

5.2.1. Identifying 'base' Plates.

The procedure for identifying the 'base' plates and their assembly sequence is the same procedure as given in 5.1. The same procedure is also used for identifying stiffener connections to the 'base' plates. The stiffeners are the 'next level' in the +Z direction. Stiffeners on the combined 'base' plates may be seen in Figure 7.

The construction process of joining the 'base' plates into a larger base and their associated stiffeners is a semi-automatic process. The plates are joined using automatic welding machine and the stiffeners are welded to the 'base' using semi-automatic welding machines. Once placed into position, the welding machine will weld a seam without further manual intervention.

5.2.2. Identifying Bulkheads

Set *B* is created from the contour matrix (13), where F_1 is the plate part type, F_2 is the height being equal to the block height and F_3 is the surface normal that is orthogonal to Z direction.

$$\|C_{i(j)}\|_{A,F(A)} = [A \times F(A)] = a_i \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ \vdots & \cdots & \cdots \\ c_{i1} & c_{i2} & c_{i3} \\ \vdots & \vdots & \cdots & \vdots \\ a_n \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ \vdots & \cdots & \vdots \\ c_{n1} & c_{n2} & c_{n3} \end{pmatrix}$$
(13)

A bulkhead satisfies the following equation,

$$c_{i(1)} = 1 \wedge c_{i(2)} = 1 \wedge c_{i(3)} = 1 \tag{14}$$

The set of all such plates that constitute bulkheads, B, is such that, $B \subset A$ and

$$B = \left\{ b \mid \bigwedge_{\substack{i=1,n\\j=1,3}} c_{ij} \right\}$$
(15)

5.2.3. Identifying Stiffeners, Plates and Flanges Connected To Bulkheads

Bulkheads have other parts welded to them, as may be seen in Figure 5. These may be other bulkheads, stiffeners, plates, brackets or flanges. In the normal process of things, brackets are often welded in their position after the assembly of the sub-assemblies into their final position, hence, brackets will not be considered at this stage.

Stiffeners, plates and flanges are often welded to a bulkhead, as may be seen in Figure 6. The flanges in Figure 6 form the rim in the aperture of the bulkhead. The other parts shown are stiffeners or plates.

Taking the Cartesian cross product of the element of the set A and the colour set B, the result of which may be seen in matrix (16). In this matrix, $\bigcup_{i=1,m} F_{Bi}$, is the set of

bulkheads as defined in (15). F_1 is stiffeners, F_2 is flanges, F_3 is plates and F_4 is bulkheads. A part, a_i is connected to a bulkhead B_j , if it satisfies $c_{ibj} = 1 \land c_{iF_1} = 1$, in which case the part is a stiffener

$$\|C_{i(j)}\|_{A,A(B)} = [A \times F(B)] = a_i \begin{bmatrix} c_{1B2} & \cdots & c_{1Bj} & \cdots & c_{1Bm} & c_{1F_1} & c_{1F_2} & c_{1F_3} & c_{1F_4} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ c_{iB2} & \cdots & c_{iBj} & \cdots & c_{iBm} & c_{iF_1} & c_{iF_2} & c_{iF_3} & c_{iF_4} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ a_n \begin{pmatrix} c_{nB2} & \cdots & c_{nBj} & \cdots & c_{nBm} & c_{nF_1} & c_{nF_2} & c_{nF_3} & c_{nF_4} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ c_{nB2} & \cdots & c_{nBj} & \cdots & c_{nBm} & c_{nF_1} & c_{nF_2} & c_{nF_3} & c_{nF_4} \end{bmatrix}$$

$$(16)$$

It needs to be noted that these stiffeners are the ones that are on the surface of the bulkhead, not the ones that intersect in any other way.

Let
$$S = \{s \mid c_{iBj} = 1 \land c_{iF_1} = 1\}$$
 be the set of all stiffeners connected to bulkhead B_j .

Let $G = \{h \mid c_{ibj} = 1 \land c_{iF_2} = 1\}$ be the set of all flanges connected to bulkhead B_j .

Let $P = \{p \mid c_{iB_i} = 1 \land c_{iF_3} = 1\}$ be the set of all plates that are connected to bulkhead B_j .

Let $H = \{h \mid c_{iBj} = 1 \land c_{iF_4} = 1\}$ be the set of all bulkheads connected to bulkhead B_j .

Once identified, stiffeners may be welded to their associated bulkheads. This process may be manual welding or machine welding, or a combination of both, depending on the length and stiffener configuration. The same can be said for any large plates that are welded to bulkheads.

Some plates are quite small and are used to cover holes to ensure that the structure is water tight. The identification of these small parts has been omitted here for the sake of brevity.

Bulkhead-to-bulkhead connections are important. They can be seen in Figure 5. As can be seen, there are 2 basic types of connections; edge-to-edge and edge-to-face. Other pigmentations are the length of the bulkhead, the total boundary box and the weight. The length should not be greater than 9 meters. The boundary box should have a width less than 3 meters. F1: edge-to-edge connection. F2: edge-to-face connection (relative to 1^{st} bulkhead), F3: length < 9 meters. F4: combined width < 3 meters. F5: weight < crane maximum. F6: Bounding box width <0.5 m for 2^{nd} bulkhead.

$$\|C_{i(j)}\|_{H,F(H)} = [H \times F(H)] = (h_i, h_n) \begin{bmatrix} c_{1(1)} & c_{1(2)} & c_{1(3)} & c_{1(4)} & c_{1(5)} & c_{1(6)} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \cdots \\ c_{i(1)} & c_{i(2)} & c_{i(3)} & c_{i(4)} & c_{i(5)} & c_{1(6)} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \cdots \\ c_{i(1)} & c_{i(2)} & c_{i(3)} & c_{i(4)} & c_{i(5)} & c_{1(6)} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \cdots \\ c_{n(1)} & c_{n(2)} & c_{n(3)} & c_{n(4)} & c_{n(5)} & c_{n(6)} \end{bmatrix}$$

$$(17)$$

An edge-to-edge connection is acceptable if $c_{i1} = 1 \land c_{i3} = 1 \land c_{i4} = 1 \land c_{i5} = 1 \land c_{i6} = 1$. An edge-to-face connection is acceptable if $c_{i2} = 1 \land c_{i3} = 1 \land c_{i4} = 1 \land c_{i5} = 1 \land c_{i6} = 1$.

From the above, a new set *N* is obtained where $N = S \cup G \cup P \cup H$; it is this set that is used to obtain the assembly sequence. The procedure is that outlined in section 5.1.



Figure 8: Images of Block Assembly

Figure 8 illustrates the assembly sequence of the block assembly. Some steps have been skipped for brevity. Referring to Figure 8, picture #1 has the 'base' plates and 1 sub-assembly. The longitudinal movement in this block is left to right in the pictures. The transverse movement is bottom left to top right. As can be seen, the sub-assemblies that have been identified are added, one-by-one. Picture #9 is the penultimate stage; the final stage can be seen in Figure 4.

6. Conclusion

The use of PST has proved to be flexible, effective and a rapid way of dealing with the multifarious parts and connection types found in off-shore rig blocks. Any additional

connection type that may have been missed can easily be included as an additional colour to the relevant set and the set operations/equations can be re-performed.

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