Game Theory-based Model for Insurance Pricing in Public-Private-Partnership Project

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Abstract

In recent years, Public-Private Partnership (PPP) as a project financial method has been used worldwide. Due to off-balance-sheet finance and limited-recourse or non-recourse, more and more commercial insurance products are needed in PPP project. The lender is risk averters comparing with the public sector and the private sector. They actually determine the type and coverage of the insurance arrangement. The private sector must satisfy their requirement before obtaining the debt. Because of the inefficiency of traditional insurance and the immaturity of the insurance market for PPP project, establishing a model for financial insurance pricing in PPP project is beneficial to the private sector and the insurer. Considering the incomplete information obtained by each party, a double auction model is established to predict the premium. The model also shows that the insurance requirement from the lender will increase the probability that trade occurs; however, it also increases the premium. The result of this paper may provide theoretic foundation and thinking logic for the private sector when negotiating the insurance arrangement with the insurer.

Keywords: financial insurance, game theory, premium, risk management,

Introduction

Necessity of Insurance from Different Point of View

PPP is a new financial method which uses private funds and management skills to provide infrastructure and public services. Off-balance-sheet finance and limited-recourse or non-recourse finance are two most important characteristics of PPP. The debt payment to the lender and the dividend payment to the sponsor mainly depend on the cash flow of the PPP project. Therefore, risk is a crucial factor in PPP project since it affects the ability of the project to repay project cost, debt service and dividend to the sponsor. If the risk has not been anticipated and properly hedged, the cash flow of the project will be affected leading

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to the default of the project.

In PPP project, special purpose vehicle (SPV) established by the private sector runs the project during the whole life of the project. Gatti (2008) indicates three basic strategies that the SPV can adopt to mitigate the impact of risk: transferring the risk by allocating them to key counterparties through operating contract, transferring the risk to professional agents (insurers) and retaining the risk. Figure 1 shows the risk level and risk mitigation strategies in PPP project.

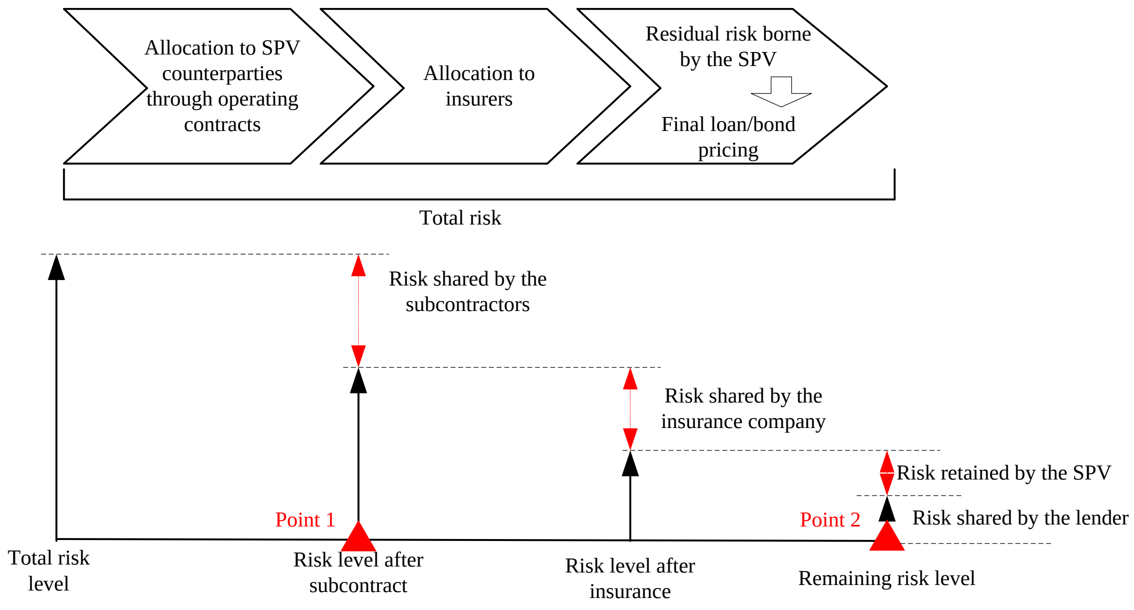

Figure 1. Risk level and risk mitigation strategies in PPP project

In PPP project, the insurable risks are transferred from the public sector to the private sector. However, the self-insurance ability or ability of retaining risk of the private sector is lower than that of the public sector due to limited free financial resources(Treasury, 2007). Therefore, more commercial insurance programs are and tend to be used in PPP project.

From the sponsor's point of view, it is in their own interest to insure against some risks. The sponsor normally will take the lead role in arranging project insurance. From the public sector's point of view, they mostly want to ensure the sustainability of the project because most PPP projects are infrastructure project. They also want to ensure that the project insurance comply with local laws. For example, workmen's compensation insurance is compulsory in some countries (Chengwing, 2008).

As for the lender, the insurance program is, in fact, an effective part of the security package (Gatti, 2008). It can be a determining factor in the lenders' approval of a PPP project package. In PPP project, the lenders have no or limited lien on the sponsor's assets outside the project vehicle. The lender must scrutinize the insurance arrangement to ensure that the cash-flow generation of the PPP project is stable and the debt can be paid in the future. Therefore the lender is the party that will determine how much and what type of insurance coverage should be purchased (Chengwing, 2008).

In general, the private sector or the SPV must satisfy the insurance requirements from the lender in order to obtain the required financing. To achieve an acceptable credit quality required by the lender, the private sector may arrange a more extensive insurance coverage. Otherwise, the lender may decrease the debt amount, increase the interest rate or even reject the debt requirement.

The role of insurance in PPP projects is not limited to risk transfer. Insurance market also provides a source of finance directly or indirectly. Project debt can be sold to a few large life insurance companies through private placements (Chengwing, 2008). To simplify the problem, insurance as a risk transfer method only is considered in this paper.

Insufficiency of Traditional Insurance

Traditional insurance, such as construction all risk and the third party insurance, is not enough to cover all risks of the PPP project. Traditional insurance can cover over 50% of the identified risks occurring during the construction and operation phase of the project (UNEP, 2007). This is insufficient for the PPP project.

Chengwing (2008) discussed the limitation of traditional insurance which can be fatal to a PPP project. It is indicated that traditional insurance is well suited to asset-based lending in which the lender's main concern is the availability of the asset when needed. Traditional insurance is focused on the physical damages and any consequential loss caused by actual physical loss. While in project-based lending, the main parties, such as the private sector and the lenders, pay more concern on the stable and sufficient revenue stream generated by the project. The revenue stream can be affected by a broad range of factors other than the physical asset, i.e. commodity price, demand, interest rate, political risk etc. Therefore, financial insurance products should be and have been used to extend the traditional insurance to focus on financial loss caused by the above described factors. Recently, insurers have become more active in covering completion risk, operating risk, off-take risk, political risk and market risk etc. (Davis, 2003).

Table 1. General summary of the insurance product

Traditional insurance	Financial insurance
Constructors' All Risk Erection All Risk	Business interruption
Third Party Liability	Residual value insurance
Force Majeure	Revenue risk mitigation
Delay in Startup Advanced Loss of Profit	Contingent capital
Property all risk insurance	Credit Delivery Guarantees (For renewable energy project)
Physical Damage Operating All Risk	Derivatives (Futures, forwards, options and swaps)
Political Risk Insurance	Warranty insurance

Source: Chengwing (2008), Gatti (2008)

The general summary of the insurance product, including traditional and financial insurance product is shown in Table 1. The aim of the table is to show the increasing trend of financial insurance, not to distinguish the two classes of insurance product.

When financial insurance is used, the risk coverage from the insurance industry is changed from construction to the whole life period of the project, from property and casualty to financial risk. When traditional insurance, financial insurance are fed into the PPP project, the economical results or the bankability of the project could be improved.

The Most Problematic Issues

The insurance market is not yet an active player in PPP projects, while the demand for PPP-oriented insurance products is growing. To some extent, there is a lack of standardized financial insurance product in the market.

The price of insurance is a most concerned element for PPP project. It is considered to be a cost to the project. HM Treasure (2007) indicated that the private sector should price the insurance in the tender document. Gatti (2008) pointed out that the pricing of the insurance package during the whole life of the project is one of the most problematic areas.

Therefore, the aim of this paper is to establish a game theory-based pricing model for financial insurance product required by the lender.

Methodology

The methodology adopted for this study is static Bayesian game theory, which is used in incomplete information games. In a game of incomplete information, at least one player is uncertain about another player's payoff function. In this paper, it is assumed that the private sector and the insurer have different valuations of the probability that insured risk will happen. These valuations are private information to each party. The payoff to each party is a function of the valuation of the probability. Therefore static Bayesian game theory is suitable for the game between the private sector and the insurer.

Gibbons (1992) have given the definition of normal form representation of n-player static Bayesian game and the definition of Bayesian Nash equilibrium. The normal-form representation of an n-player static Bayesian game can be denoted by G={$A_1,\ldots,A_n$; $T_1,\ldots T_n$; $p_1,\ldots,p_n$; $u_1,\ldots,u_n$}. $A_1,\ldots,A_n$ are the players' action space, $T_1,\ldots T_n$ are their type spaces, $p_1,\ldots,p_n$ are their beliefs spaces, and $u_1,\ldots,u_n$ are their payoff functions. The action choices of the players in a game is $(a_1,\ldots,a_n)$ which is a subset of action space $A_1,\ldots,A_n$. Player i's type, t_i, is private information known by player i, which determines player i's payoff function, $u_i(a_1,\ldots,a_n; t_i)$ and is a member of the set of possible types, T_i. Play i's uncertainty about the n-1 other players' possible types, t_{-i}, is described by player i's belief $p_i(t_{-i} \mid t_i)$, given i's own type, t_i. In the static Bayesian game, a strategy for player i is a function $s_i(t_i)$, where for each type t_i in T_i, $s_i(t_i)$ specifies the action from the feasible set A_i. In the static Bayesian game, the strategies $s^*=(s_1^*,\ldots, s_n^*)$ are a Bayesian Nash equilibrium if for each player i and for each of i's types t_i in T_i, $s_i^*(t_i)$ solves Eq.(1)

$$\max_{a_i \in A_i} \sum_{t_{-i} \in T_i} u_i(s_i^*(t_1),\ldots,s_{i-1}^*(t_{i-1}),a_i,s_{i+1}^*(t_{i+1}),\ldots,s_n^*(t_n);t)\,p_i(t_{-i} \mid t_i) \qquad (1)$$

Eq.(1) means that for each player i, a_i is player i's best response (action) to the strategies specified for the n-1 other players ($s_i^*(t_1),\ldots, s_{i-1}^*(t_{i-1}), s_{i+1}^*(t_{i+1}),\ldots,s_n^*(t_n)$) and player i's belief about the uncertainty that the n-1 other players' possible types is t_{-i}, given i's own type, t_i. Therefore no player wants to change his or her strategy.

Gibbons (1992) discussed a case in which a buyer and a seller each have private information about their valuations of the good. The case is named as a double auction game which is an incomplete information game. In this paper, the insurer is the seller and the private sector is the buyer. Each of them has private information about their valuations of the probability that the insured risk event will happen.

Result-Model Setup

Assumptions

As previously discussed, the lender is the party that will determine how much and what type of insurance coverage should be purchased by the private sector. In fact, the lender will do bankability analysis before lending the debt to the private sector. If they belief that some kinds of risk are too large which will easily lead the project in default, they probably will require the private sector to make insurance arrangement to hedge them. It is assumed that whether the private sector can satisfy the requirement from the lender determines the debt terms, such as debt amount, interest rate, debt term and reserve account etc. Suppose the difference of net present value of the project causing by debt terms is ΔNPV.

The Double Auction Model

The players are the private sector and the insurer. There are two different conditions for this game. When the risk coverage is fixed, they will negotiate about the premium. While when the premium is fixed, they will negotiate about the risk coverage. In this paper, the first condition will be discussed. Therefore, their strategy is premium.

In this game, the insurer is the seller who sells the insurance product to the private sector. The private sector is the buyer of the insurance product. In this game, it is assumed that the insurer and the private sector simultaneously offer the premium. The premium from the insurer is p_s and the premium from the private sector is p_b. If $p_b \geq p_s$, then the trade occurs at price $p = (p_s + p_b)/2$; if $p_b < p_s$, then no trade occurs. If the trade can occur, there actually exists a bargaining game. The buyer and the seller will bargain over the premium. The premium usually is finalized in the middle of p_s and p_b. To simplify this problem, it is assumed that $p = (p_s + p_b)/2$.

According to discounted cash flow model which was developed by Myers and Cohn (1987) for the 1982 Massachusetts automobile rate hearings, the basic formulation for insurance pricing is shown in Eq.(2) (D'Arcy and Garven, 1990).

$$P = PV(LE) + PV(UWPT) + PV(IBT) \tag{2}$$

In Eq.(2), P means the premiums; LE means losses and underwriting expenses; $UWPT$ means tax generated on underwriting income; IBT means tax generated on income from the investment balance; $PV(\cdot)$ is the present value operator (D'Arcy and Garven, 1990). In this model, the premium is equated with cash flows for losses, expenses and taxes. In general, insurers will collect premium income either when the policy is written or in installments over the policy period. However, the cost of the loss, expenses and tax are all over the policy period. The above considerations are the reason for using present value operator. For simplicity, all values in this paper are considered to be present value

As for insurance pricing model, the underwriting aspect of the insurance and the investment aspect of the insurance should be considered together(D'Arcy and Garven, 1990). However, insurance as a risk transfer method is only considered in this paper. There is no need to consider the investment aspect of the insurance. The expected cost of the insurance product is presented in Eq. (3).

$$\begin{aligned} E(Cost_s) &= E + E(L) + T_s \\ &= \gamma \cdot p_s + \alpha_s \cdot C + t(p_s - \gamma \cdot p_s - \alpha_s \cdot C) \end{aligned} \tag{3}$$

In Eq. (3), E means expense; $E(L)$ means the expected losses; T_s means tax generated on underwriting income; γ means expenses' coefficient as to premium; α_s means the probability predicted by the insurer that the risk event will happen; C means the compensation amount; t means the corporate tax rate. The insurer can obtain the values of α_s and γ by regression analysis based on historical data. In this paper, it is assumed that γ is a fixed value. Therefore α_s is private information known by the insurer. If the insurer signs the insurance contract for premium p, then the insurer's utility is $p - E\,(Cost_s)$. If there is no trade, the utility is zero.

The expected income of the private sector is presented in Eq. (4).

$$\begin{aligned} E(Income_b) &= E(C) + T_b + \Delta NPV \\ &= \alpha_b \cdot C + t \cdot (p_b - \alpha_b \cdot C) + \Delta NPV \end{aligned} \tag{4}$$

In Eq. (4), $E(C)$ means the expected compensation; α_b means the predicted probability by the private sector that the risk event will happen. The private sector has more information about the project than the insurer. They can use Monte Carlo Simulation or Stochastic method to predict the probability that the risk event will happen. Therefore, α_b is private information known by the private sector. If the private sector signs the insurance contract for premium p, then the private sector's utility is $E(Income_b)$ - p. If there is no trade, the private sector's utility is zero.

In this static Bayesian game, a strategy for the private sector is a function $p_b\,(\alpha_b)$ specifying the premium the private sector will offer for each of its possible valuations of probability. Likewise, a strategy for the seller is a function $p_s\,(\alpha_s)$ specifying the premium the insurer will offer for each of its possible valuations. It is assumed that the probabilities are drawn from independent uniform distribution on [0, 1]. A pair of strategies $\{p_b\,(\alpha_b), p_s\,(\alpha_s)\}$ is a Bayesian Nash Equilibrium if the following two conditions hold.

For each α_s in [0, 1], $p_s\,(\alpha_s)$ solves

$$
\begin{aligned}
&\max_{p_s}[p - E(Cost_s)]\Pr ob\{p_b(\alpha_b) \geq p_s\} \\
&= \max_{p_s}[p - \gamma \cdot p - \alpha_s \cdot C - t(p - \gamma \cdot p - \alpha_s \cdot C)]\Pr ob\{p_b(\alpha_b) \geq p_s\} \\
&= \max_{p_s}[(1-\gamma) \cdot p - \alpha_s \cdot C] \cdot (1-t) \cdot \Pr ob\{p_b(\alpha_b) \geq p_s\} \\
&= \max_{ps}[(1-\gamma) \cdot \frac{p_s + E[p_b(\alpha_b) | p_b(\alpha_b) \geq p_s]}{2} - \alpha_s \cdot C] \cdot (1-t) \cdot \Pr ob\{p_b(\alpha_b) \geq p_s\}
\end{aligned}
\tag{5}
$$

Where $E\ [p_b\ (\alpha_b)\ |\ p_b(\alpha_b) \geq p_s]$ is the expected price the private sector will offer, conditional on the offer being greater than the insurer's offer of p_s.

For each α_b in [0, 1], $p_b\ (\alpha_b)$ solves

$$
\begin{aligned}
&\max_{p_b}[E(Income_b) - p]\Pr ob\{p_b \geq p_s(\alpha_s)\} \\
&= \max_{p_b}[\alpha_b \cdot C + t \cdot (p - \alpha_b \cdot C) + \Delta NPV - p]\Pr ob\{p_b \geq p_s(\alpha_s)\} \\
&= \max_{p_b}\{(1-t) \cdot [\alpha_b \cdot C - \frac{p_b + E[p_s(\alpha_s) | p_b \geq p_s(\alpha_s)]}{2}] + \Delta NPV\}\Pr ob\{p_b \geq p_s(\alpha_s)\}
\end{aligned}
\tag{6}
$$

Where $E\ [p_s\ (\alpha_s)\ |\ p_b \geq p_s\ (\alpha_s)]$ is the expected price the insurer will demand, conditional on the demand being less than the private sector's offer of p_b.

Suppose the private sector's strategy is $p_b\ (\alpha_b) = x_b + y_b \cdot \alpha_b$. Then p_b is uniformly distributed on $[x_b, x_b + y_b]$, so Eq. (5) becomes Eq. (7).

$$
\max_{p_s}[(1-\gamma) \cdot \frac{p_s + \frac{x_b + y_b + p_s}{2}}{2} - \alpha_s \cdot C] \cdot (1-t) \cdot \frac{x_b + y_b - p_s}{y_b} \tag{7}
$$

The first-order condition for Eq. (7) yields Eq. (8).

$$
p_s = \frac{1}{3}(x_b + y_b) + \frac{2 \cdot C}{3(1-\gamma)} \cdot \alpha_s \tag{8}
$$

Thus, if the insurer plays a linear strategy, then the private sector's best response is also linear. Analogously, supposing the private sector's strategy is $p_s\ (\alpha_s) = x_s + y_s \cdot \alpha_s$. Then is uniformly distributed on $[x_s, x_s + y_s]$, so Eq. (6) becomes Eq. (9).

$$
\max_{p_b}[(1-t) \cdot (\alpha_b \cdot C - \frac{p_b + \frac{p_b + x_s}{2}}{2}) + \Delta NPV] \cdot \frac{p_b - x_s}{y_s} \tag{9}
$$

The first-order condition for Eq. (9) yields Eq. (10).

$$p_b = \frac{2}{3}(\frac{x_s}{2} + \Delta NPV^*) + \frac{2}{3} \cdot C \cdot \alpha_b \tag{10}$$

Note: $\Delta NPV^* = \Delta NPV / (1-t)$

If the player's linear strategies are to be best response to each other, Eq. (8) and Eq. (10) imply Eq. (11).

$$\begin{cases} x_s = \frac{1}{3}(x_b + y_b) \\ y_s = \frac{2C}{3(1-\gamma)} \\ x_b = \frac{2}{3}(\frac{x_s}{2} + \Delta NPV^*) \\ y_b = \frac{2}{3}C \end{cases} \tag{11}$$

Therefore, the linear equilibrium strategies are shown in Eq.12 and Eq. 13.

$$p_s(\alpha_s) = \frac{1}{4}\Delta NPV^* + \frac{1}{4}C + \frac{2C}{3(1-\gamma)} \cdot \alpha_s \tag{12}$$

$$p_b(\alpha_b) = \frac{3}{4}\Delta NPV^* + \frac{1}{12}C + \frac{2C}{3} \cdot \alpha_b \tag{13}$$

Discussion

Premium Affected by Debt Requirements

As can be seen from Eq. (12) and Eq. (13), the premium will increase with the ΔNPV^*. As for the private sector, they are willing to offer higher premium if the insurance arrangement can satisfy the requirement from the lender and improve the bankability of the project. As for the insurer, they will require higher premium if they know that the insurance arrangement will improve the debt terms.

Having the information that insurance product is required by the lender makes the insurer better off. However, it makes the private sector worse off because the premium is increased. Therefore, the private sector should protect this kind of information.

Trade Requirement

As previous stated, trade occurs in the double auction if and only if $p_b \geq p_s$. Manipulating Eq. (12) and Eq. (13) shows that trade occurs in the linear equilibrium if and only if the following equation holds.

$$\alpha_b \geq \frac{\alpha_s}{(1-\gamma)} + \frac{1}{4} - \frac{3\Delta NPV^*}{4C} \tag{13}$$

The trade area is shown in Figure 2. As can be seen from Figure 2, when the tax deducted difference of the net present value is small comparing to the compensation amount which makes 1/4 -3 $\Delta NPV^*/4C > 0$, the trade area is above the line $\alpha_b = \alpha_s/(1-\gamma)$. When ΔNPV^* increases, the trade area increases which may be below the line $\alpha_b = \alpha_s / (1-\gamma)$. It implies that the insurance requirement from the lender increases the probability that trade occurs.

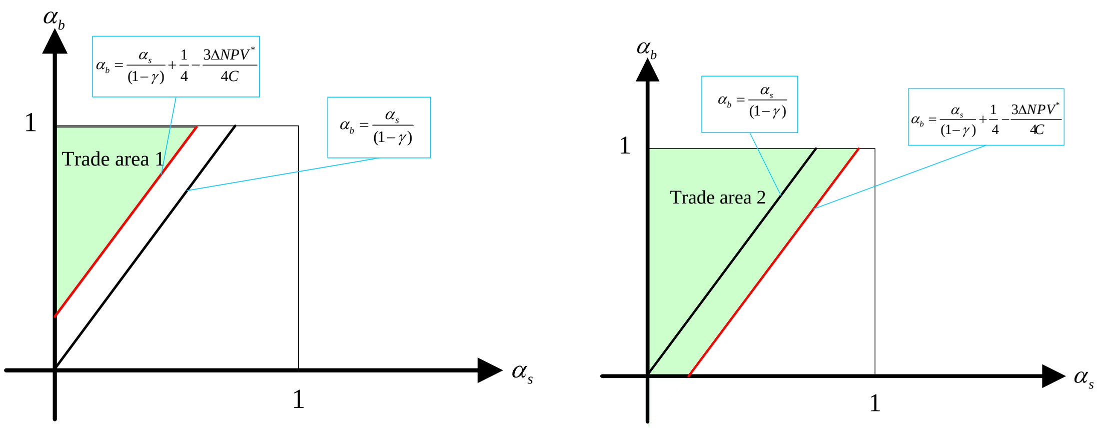

Figure 2. Trade area

Probability Prediction

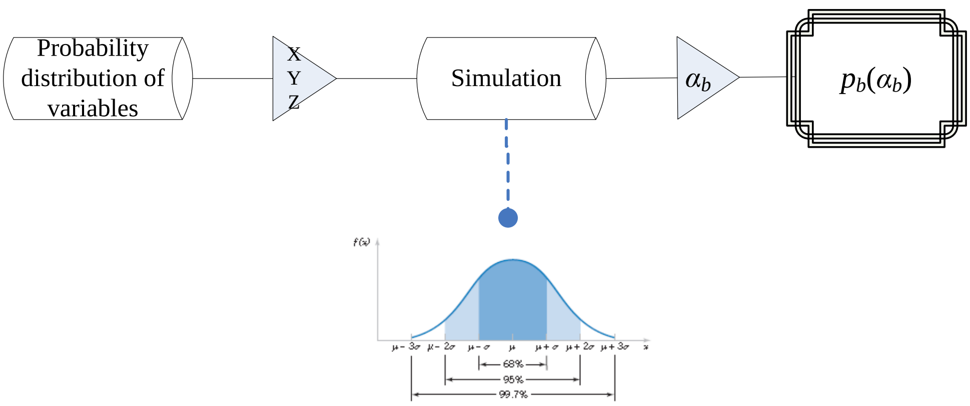

Figure 3. The steps of probability prediction

The insurer assign insurance contract with a lot of project. They can predict the probability α_s through regression analysis based on historical data. As for the private sector, PPP project is one time project. They can use Monte Carlo Simulation method to predict the probability α_b.

In reality, multivariable are uncertain and they may happen concurrently. Monte Carlo Simulation method is a class of computational algorithms that rely on repeated random sampling to compute their results(Mum, 2006). Figure 4 shows the steps of probability prediction.

Conclusion

Insurance is an important risk mitigation strategy for PPP project. Due to the characteristics of off-balance-sheet finance and limited-recourse or non-recourse, the lender has requirements about the type and coverage of the insurance arrangement. Considering these requirements from the lender and incomplete information between the private sector and insurer, this paper established a double auction model based on incomplete information game theory for financial insurance pricing in PPP project. The model shows that the insurance requirements from the lender will increase the probability that trade occurs; however, it also increases the premium. The result of this paper may provide theoretic foundation and thinking logic for the private sector when negotiating the insurance arrangement with the insurer.

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